Psychological Test and Assessment Modeling, Volume 60, 2018 (1), 29-32

Guest Editorial
Rater effects: Advances in item response modeling of human ratings – Part II

Thomas Eckes[1]

The papers in Part I of this special issue dealt with rater effects from the perspective of two-facet IRT modeling (Wu, 2017), multilevel, hierarchical rater models (Casabianca & Wolfe, 2017), and nonparametric Mokken analysis (Wind & Engelhard, 2017). Part II includes papers that probe further into the complex nature of human ratings within the context of performance assessment, highlighting the benefits and challenges of examining rater effects from different angles and with different levels of detail.

In the first paper, entitled "A tale of two models: Psychometric and cognitive perspectives on rater-mediated assessments using accuracy ratings", George Engelhard, Jue Wang, and Stefanie A. Wind elaborate on the need to bring together psychometric and cognitive perspectives in order to gain a deeper understanding of rater-mediated assessments (Engelhard, Wang, & Wind, 2018). Whereas psychometric perspectives have long dominated the field, cognitive perspectives with their specific focus on the study of human categorization, judgment, and decision making in assessment contexts have only recently attracted more attention (Bejar, 2012). In the paper, Engelhard et al. build on Brunswik's (1952) lens model as a cognitive approach and conceptually link this model to many-facet Rasch measurement (MFRM; Linacre, 1989). Their study is situated within an external frame of reference, that is, a group of experts provided criterion ratings that were compared to operational ratings to obtain rating accuracy data. Using the Rater Accuracy Model (RAM; Engelhard, 1996), the authors construct measures for the accuracy of individual raters in a writing assessment and analyze which examinee performances and writing domains, respectively, were difficult to rate accurately.

In the second paper, entitled "Modeling rater effects using a combination of generalizability theory and IRT", Jinnie Choi and Mark R. Wilson adopt a generalized linear latent and mixed model (GLLAMM) approach to combine what many researchers and assessment specialists have considered fundamentally different methods to study rating quality (Choi & Wilson, 2018). As discussed in the Editorial to Part I (Eckes, 2017),

[1] *Correspondence concerning this article should be addressed to:* Thomas Eckes, PhD, TestDaF Institute, University of Bochum, Universitätsstr. 134, 44799 Bochum, Germany; email: thomas.eckes@testdaf.de

generalizability theory (GT; e.g., Brennan, 2001) and IRT are commonly thought to represent diverging research traditions. Simply put, GT, being rooted in classical test theory and analysis of variance, focuses on observed test scores, whereas IRT focuses on item responses and how they relate to the ability being measured (Brennan, 2011; Linacre, 2001). Against this background, Choi and Wilson demonstrate that much is to be gained from integrating both approaches into a logistic mixed model that allows not only to estimate random variance components and generalizability coefficients for examinees, items, and raters, but also to construct individual examinee, item, and rater measures as known from IRT applications (see also Robitzsch & Steinfeld, 2018a). Further advantages of the combined approach refer to its flexibility regarding the analysis of multidimensional and/or polytomous item response data and the graphical presentation of predicted individual random effects in modified Wright maps.

In the third paper, entitled “Comparison of human rater and automated scoring of test takers’ speaking ability and classification using item response theory”, Zhen Wang and Yu Sun provide a detailed look at the performance of an automated scoring system for spoken responses (Wang & Sun, 2018). Specifically, the authors use the automated scoring engine SpeechRater, developed at Educational Testing Service (ETS), to score examinee performances on the speaking section of an English language assessment, and compare the scores from SpeechRater to scores assigned by human raters. Wang and Sun consider a range of scoring scenarios representing various combinations of SpeechRater and human ratings, such as human rater only, SpeechRater only, and differential weighting of SpeechRater and human rater contributions to the final scores. Building on structural equation modeling and IRT scaling (GPCM; Muraki, 1992), the authors find pronounced differences between the results obtained for each of these scenarios, indicating that automated scores and human rater scores of spoken responses do not reflect the same underlying construct.

The final paper, entitled "Item response models for human ratings: Overview, estimation methods, and implementation in R" by Alexander Robitzsch and Jan Steinfeld, first provides a brief introduction to IRT models for human ratings, including many-facet rater models based on partial credit, generalized partial credit, and graded response modeling approaches, as well as generalized many-facet rater models, covariance structure models, and hierarchical rater models (Robitzsch & Steinfeld, 2018a). The authors go on to present various maximum likelihood and Bayesian methods of estimating parameters for each of these models. Following a thoughtful discussion of how to choose between the different models, Robitzsch and Steinfeld illustrate the practical model use with a real data set. For this purpose, they draw on three different, highly versatile R packages for estimating IRT models for multiple raters: "immer" (Item Response Models for Multiple Ratings; Robitzsch & Steinfeld, 2018b), "sirt" (Supplementary Item Response Theory Models; Robitzsch, 2018), and "TAM" (Test Analysis Modules; Robitzsch, Kiefer, & Wu, 2018). The findings from these analyses are compared with linear mixed effects models implemented in the “lme4” package (Bates, Mächler, Bolker, & Walker, 2015). For each data analysis, the authors provide excerpts from the R syntax along with detailed explanations in order to guide readers in how to best use the R packages with their own research.

Taken together, the psychometric approaches, models, and analyses documented in Parts I and II provide new insights into rater effects across a wide range of assessment contexts. It seems evident that item response modeling has made much progress both in terms of detecting rater effects and mitigating or even correcting at least part of the negative impact these effects have on the validity and fairness of human ratings. May these advances stimulate not only future research in the field, but also inform practical decisions regarding the design, implementation, and evaluation of rater-mediated assessments.

References

Bates, D., Mächler, M., Bolker, B., & Walker, S. (2015). Fitting linear mixed-effects models using lme4. *Journal of Statistical Software*, *67*(1).

Bejar, I. I. (2012). Rater cognition: Implications for validity. *Educational Measurement: Issues and Practice*, *31*(3), 2–9.

Brennan, R. L. (2001). *Generalizability theory*. New York, NY: Springer.

Brennan, R. L. (2011). Generalizability theory and classical test theory. *Applied Measurement in Education*, *24*, 1–21.

Brunswik, E. (1952). *The conceptual framework of psychology*. Chicago, IL: University of Chicago Press.

Casabianca, J. M., & Wolfe, E. W. (2017). The impact of design decisions on measurement accuracy demonstrated using the hierarchical rater model. *Psychological Test and Assessment Modeling*, *59*(4), 471–492.

Choi, J., & Wilson, M. R. (2018). Modeling rater effects using a combination of generalizability theory and IRT. *Psychological Test and Assessment Modeling*, *60*(1), 53–80.

Eckes, T. (2017). Rater effects: Advances in item response modeling of human ratings – Part I (Guest Editorial). *Psychological Test and Assessment Modeling*, *59*(4), 443–452.

Engelhard, G. (1996). Evaluating rater accuracy in performance assessments. *Journal of Educational Measurement*, *33*, 56–70.

Engelhard, G., Wang, J., & Wind, S. A. (2018). A tale of two models: Psychometric and cognitive perspectives on rater-mediated assessments using accuracy ratings. *Psychological Test and Assessment Modeling*, *60*(1) 33–52.

Linacre, J. M. (1989). *Many-facet Rasch measurement*. Chicago, IL: MESA Press.

Linacre, J. M. (2001). Generalizability theory and Rasch measurement. *Rasch Measurement Transactions*, *15*, 806–807.

Muraki, E. (1992). A generalized partial credit model: Application of an EM algorithm. *Applied Psychological Measurement*, *16*, 159–176.

Robitzsch, A. (2018). Package 'sirt': Supplementary item response theory models (Version 2.5) [Computer software and manual]. Retrieved from https://cran.r-project.org/web/packages/sirt/index.html

Robitzsch, A., Kiefer, T., & Wu, M. (2018). Package 'TAM': Test analysis modules (Version 2.9) [Computer software and manual]. Retrieved from https://cran.r-project.org/web/packages/TAM/index.html

Robitzsch, A., & Steinfeld, J. (2018a). Item response models for human ratings: Overview, estimation methods and implementation in R. *Psychological Test and Assessment Modeling*, *60*(1), 101–138.

Robitzsch, A., & Steinfeld, J. (2018b). Package 'immer': Item response models for multiple ratings (Version 1.0) [Computer software and manual]. Retrieved from https://cran.r-project.org/web/packages/immer/index.html

Wang, Z., & Sun, Y. (2018). Comparison of human rater and automated scoring of test takers' speaking ability and classification using item response theory. *Psychological Test and Assessment Modeling*, *60*(1), 81–100.

Wind, S. A., & Engelhard, G. (2017). Exploring rater errors and systematic biases using adjacent-categories Mokken models. *Psychological Test and Assessment Modeling*, *59*(4), 493–515.

Wu, M. (2017). Some IRT-based analyses for interpreting rater effects. *Psychological Test and Assessment Modeling*, *59*(4), 453–470.

Psychological Test and Assessment Modeling, Volume 60, 2018 (1), 33-52

A tale of two models: Psychometric and cognitive perspectives on rater-mediated assessments using accuracy ratings

George Engelhard, Jr[1], *Jue Wang*[2], *& Stefanie A. Wind*[3]

Abstract

The purpose of this study is to discuss two perspectives on rater-mediated assessments: psychometric and cognitive perspectives. In order to obtain high quality ratings in rater-mediated assessments, it is essential to be guided by both perspectives. It is also important that the specific models selected are congruent and complementary across perspectives. We discuss two measurement models based on Rasch measurement theory (Rasch, 1960, 1980) to represent the psychometric perspective, and we emphasize the Rater Accuracy Model (Engelhard, 1996, 2013). We build specific judgment models to reflect the cognitive perspective of rater scoring processes based on Brunswik's Lens model framework. We focus on differential rater functioning in our illustrative analyses. Raters who possess inconsistent perceptions may provide different ratings, and this may cause various types of inaccuracy. We use a data set that consists of the ratings of 20 operational raters and three experts of 100 essays written by Grade 7 students. Student essays were scored using an analytic rating rubric for two domains: (1) idea, development, organization, and cohesion; as well as (2) language usage and convention. Explicit consideration of both psychometric and cognitive perspectives has important implications for rater training and maintaining the quality of ratings obtained from human raters.

Keywords: Rater-mediated assessments, Rasch measurement theory, Lens model, Rater judgment, Rater accuracy

[1]*Correspondence concerning this article should be addressed to:* George Engelhard, Jr., Ph.D., Professor of Educational Measurement and Policy, Quantitative Methodology Program, Department of Educational Psychology, 325W Aderhold Hall, The University of Georgia, Athens, Georgia 30602, U.S.A. email: gengelh@uga.edu

[2]The University of Georgia

[3]The University of Alabama

Rater-mediated performance assessments are used in many countries around the world to measure student achievement in a variety of contexts. For example, Lane (2016) has noted: "performance assessments that measure critical thinking skills are considered to be a valuable policy tool for improving instruction and student learning in the 21st century" (p. 369). Performance assessments have been used to measure proficiency in writing (Wind & Engelhard, 2013), first and second languages (Eckes, 2005; Wind & Peterson 2017), teaching (Engelhard & Myford, 2010), and student achievement in many other areas, such as music education (Wesolowski, Wind, & Engelhard, 2016).

A unique feature of performance assessments is that they require human raters to interpret the quality of a performance using a well-developed rating scale. Performance assessments can be meaningfully viewed as rater-mediated assessments because the ratings modeled in our psychometric analyses are directly obtained from human judges (Engelhard, 2002). One of the critical concerns for rater-mediated assessments is how to evaluate the quality of judgments obtained from raters. Raters may bring a variety of potential systematic biases and random errors to the judgmental tasks that may unfairly influence the assignment of ratings. As pointed out by Guilford (1936), "Raters are human and they are therefore subject to all of the errors to which humankind must plead guilty" (p. 272). However, good quality control and rater training can minimize the biases and errors.

In this study, we argue that two complementary perspectives are needed in order to evaluate the quality of rater judgments: (1) a measurement model and (2) a model of human judgment and cognition. Focusing on the role of these perspectives, we consider the following questions:

- What psychometric perspectives can be used to evaluate ratings in rater-mediated assessments?
- What cognitive perspectives can provide guidance on how to model judgments obtained in rater-mediated assessments?
- How can we connect these two theoretical perspectives to improve rater-mediated assessments?

Figure 1 provides a conceptual model representing our view of the connections between psychometric and cognitive perspectives on rater-mediated assessments. The psychometric and cognitive perspectives provide the base of a triangle that supports the development and maintenance of rater-mediated assessments. It is our view that the vertices in this triangle should be viewed together, and a major thesis of this study is that current research on raters and judgments do not go far enough in explicitly considering these connections.

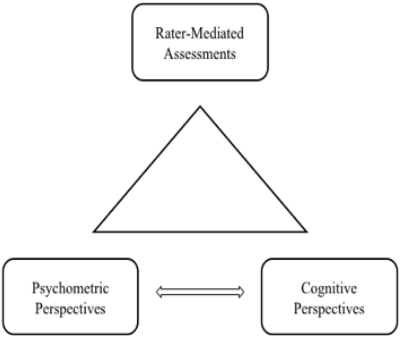


Figure 1:
Conceptual model for rater-mediated assessments

What psychometric perspectives can be used to evaluate ratings in rater-mediated assessments?

In evaluating the quality of ratings, there have been several general perspectives. These psychometric perspectives can be broadly classified into test score and scaling traditions (Engelhard, 2013). Many of the current indices used in operational testing to evaluate ratings are based on the test score tradition; for example, rater agreement indices, intraclass correlations, kappa coefficients, and generalizability coefficients (Cronbach, Gleser, Nanda, & Rajaratnam, 1972; Johnson, Penny & Gordon, 2009; von Eye, & Mun, 2005). It is safe to say that most operational performance assessment systems report the percentage of exact and adjacent category usage for operational raters. All of these models within the test score tradition treat the observed ratings as having categories with equal width. In other words, the ratings are modeled as equal intervals by using sum scores.

Ratings can also be evaluated using measurement models based on the scaling tradition (Engelhard, 2013). In the scaling tradition, the structure of rating categories is parameterized with category coefficients (i.e., thresholds). Thresholds that define rating categories are not necessarily of equal width (Engelhard & Wind, 2013). The most common IRT models for rating scale analysis include the Partial Credit Model (Masters, 1982), the Rating Scale Model (Andrich, 1978), the Generalized Partial Credit Model (Muraki, 1992), and the Graded Response Model (Samejima, 1969). The Many-Facet Rasch model (Linacre, 1989) specifically adds a rater parameter, and this model is widely used in the detection of rater effects. The Many-Facet Rasch model is a generalized form of the Rasch model that was specifically designed for rater-mediated assessments (Eckes, 2015). There are also several other rater models, such as the hierarchical rater model (Casabiaca, Junker, & Patz, 2016), that have been proposed. It is beyond the scope of this study to describe in detail other models for ratings, and we recommend Nering and Ostini (2010) for interested readers.

All of the psychometric perspectives described up to this point model the observed ratings assigned by raters. Engelhard (1996) proposed another approach based on accuracy ratings (Wolfe, Jiao, & Song, 2014). Accuracy ratings represent the distances between criterion ratings and operational ratings. For instance, criterion ratings are assigned by an expert rater or a group of expert raters. The observed ratings assigned by well-trained operational raters are referred to as operational ratings. The differences between these operational ratings and criterion ratings reflect the accuracy of operational rater judgments on each performance. Engelhard (1996) put forward an equation for calculating accuracy ratings. Since accuracy ratings reflect the distance between operational ratings and criterion ratings, we call them *direct measures* of rater accuracy. On the other hand, observed operational ratings are viewed as *indirect measures* for rater accuracy. Due to this difference, we use the term *Rater Accuracy Model*s (RAM) to label the Rasch models that examine accuracy ratings as the dependent variable on which individual raters, performances, and other facets can be measured. We present two lens models for observed operational ratings and accuracy ratings correspondingly.

Scholars have used the term *rater accuracy* in numerous ways to describe a variety of rating characteristics, including agreement, reliability, and model-data fit (Wolfe & McVay, 2012). In these applications, rater accuracy is used as a synonym for ratings with desirable psychometric properties. RAM provides a criterion-referenced perspective on rating quality that can be used to directly describe and compare individual raters, performances, and other facets in the assessment system with a focus on rater accuracy. From criterion-referenced perspective, the RAM provides a more specific definition and clear interpretation of rater accuracy. Furthermore, the criterion-referenced approach emphasizes the evaluation of rater accuracy using accuracy ratings as direct measures. These accuracy ratings can be coupled with a lens model to guide rater training and diagnostic activities during scoring.

We summarize five sources of inaccuracy due to differences among rater judgments in Table 1. First, we view *rater inaccuracy* as a tendency to consistently provide biased ratings. Second, *halo inaccuracy* or domain inaccuracy refers to the situations that raters fail to distinguish among different domains on an analytic scoring rubric when evaluating student performances. Wang, Engelhard, Raczynski, Song, and Wolfe (2017) observed this phenomenon that some raters tended to provide adjacent scores for two distinct domains of writing. Third, when raters use the rating scale in an idiosyncratic fashion, it leads to *response set inaccuracy* such that ratings are not consistent toward the *benchmarks* used as the basis for criterion ratings. Specifically, person benchmarks refer to the pre-calibrated performances (e.g., students' essays) that are used to evaluate raters' scoring proficiency. Fourth, *score range inaccuracy* occurs when ratings have less or more variation than expected based on the measurement model. Lastly, if raters interpret other facets differentially, *interaction effects* may appear in rater inaccuracy. It should also be noted that the focus (i.e., individual raters versus rater groups) yields different questions and conclusions related to rater inaccuracy. These sources of rater inaccuracy can guide researchers in identifying possible sources of rater inaccuracy with the use of RAM or other psychometric models.

Table 1:

Sources of Rater Inaccuracy

Definitions	**Focus**	
	Individual Raters	Rater Group
1. Rater Inaccuracy: The tendency on the part of raters to consistently provide higher or lower ratings than warranted based on known person benchmarks.	How accurate is each rater? Where is the rater located on the Wright map for accuracy?	Are the differences in rater accuracy significant? Can the raters be considered of equivalent accuracy?
2. Halo inaccuracy (domain inaccuracy): Rater fails to distinguish between conceptually distinct and independent domains on person benchmarks.	Is the rater distinguishing between conceptually distinct domains?	Are the raters distinguishing among the domains?
3. Response set inaccuracy: Rater interprets and uses rating scale categories in an idiosyncratic fashion.	Is the rater using the rating scale as intended?	Are the raters using the rating scales as intended?
4. Score Range Inaccuracy: More or less variation in accuracy ratings of benchmarks. Raters do not differentiate between person benchmarks on the latent variable.	How well did each rater differentiate among the benchmarks?	Did the assessment system lead to the identification of meaningful differences between the benchmarks?
5. Inaccuracy interaction effects: Facets in the measurement model are not interpreted additively.	Is the rater interpreting and using the facets accurately?	Are the facets invariant across raters?

Note. Person benchmarks represent the criterion performances (e.g., essays with ratings assigned by experts) used to evaluate rater accuracy.

There have been several recent applications of the RAM that reflect different measurement frameworks and contexts. For example, Engelhard (1996) adapted the Rasch model for examining rater accuracy in a writing assessment. Wesolowski and Wind (in press) as well as Bergin, Wind, Grajeda, and Tsai (2017) used the distance between operational and expert ratings as the dependent variable in a Many-Facet Rasch model to evaluate rater accuracy in music assessments and teacher evaluations, respectively. Another example is Patterson, Wind, and Engelhard (2017) who incorporated criterion ratings into signal detection theory for evaluating rating quality. Finally, Wang, Engelhard, and Wolfe (2016) have used accuracy ratings with an unfolding model to examine rater accuracy.

What cognitive perspectives can provide guidance on how to model judgments obtained in rater-mediated assessments?

> *The simple beauty of Brunswik's lens model lies in recognizing that the person's judgment and the criterion being predicted can be thought of as two separate functions of cues available in the environment of the decision.*
>
> *(Karelaia and Hogarth, 2008, p. 404)*

Cognitive psychology (Barsalou, 1992) offers a variety of options for considering judgment and decision-making tasks related to rater-mediated assessments. Cooksey (1996) describes 14 theoretical perspectives on judgment and decision making that can be potential models for examining the quality of judgments in rater-mediated assessments. Within educational settings, there was a special issue of *Educational Measurement: Issues and Practice* devoted to rater cognition (Leighton, 2012). There are many promising areas for future research on rater cognition and rater judgments (Lane, 2016; Myford, 2012; Wolfe, 2014).

Although there are numerous potential models of human judgment that may be useful guides for monitoring rating quality, the underlying model of judgmental processes used here is based on Brunswik's (1952) lens model. Lens models have been used extensively used across social science research contexts to examine human judgments. For example, there are two important meta-analyses of research organized around lens models. First, Karelaia and Hogarth (2008) conducted a meta-analysis of five decades of lens model studies (N=249) that included a variety of task environments. More recently, Kaufmann, Reips, and Wittmann (2013) conducted a meta-analysis based on 31 lens model studies including applications from medicine, business, education, and psychology. An important resource for recent work on lens models is the website of the Brunswik Society (http://www.brunswik.org/), which provides yearly abstracts of current research utilizing a lens model framework.

Brunswik (1952, 1955a, 1955b, 1956) proposed a new perspective in psychology called probabilistic functionalism (Athanasou & Kaufmann, 2015; Postman & Tolman, 1959). An important aspect of Brunswik's research was the concept of a lens model (Hammond, 1955; Postman & Tolman, 1959). The structure of Brunswik's lens models varied over time and application areas. Figure 2 presents a lens model for perception proposed by Brunswik (1955a). In this case, a person utilizes a set of cues (i.e., proximal-peripheral cues) to generate a response (i.e., central response). The accuracy of a person's response can be evaluated by its relationship to the distal variable, which is called functional validity. Ecological validities represent the relationships between the distal variable and the cues, while utilization validities reflect the relationship between the cues and the central response. In both cases, higher values of correspondence are viewed as evidence of validity. It is labeled a *lens model* because it resembles the way light passes through a lens defined by cues.

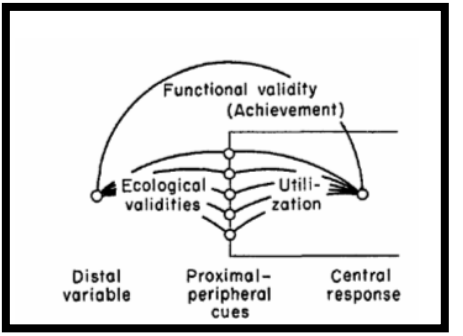


Figure 2:

Lens model for perception constancy (Adopted from Brunswik (1955a, p. 206)

In rater-mediated assessments, the accuracy of a rater's response (i.e., observed rating) is evaluated by its correspondence to or relationship with the latent variable (i.e., distal variable). Engelhard (1992, 1994, 2013) adapted the lens model as a conceptual framework for rater judgments in writing assessment. Figure 3 provides a bifocal perspective on rater accuracy in measuring writing competence. We refer to Figure 3 as *Lens Model I,* where the basic idea is that the latent variable — writing competence — is made visible through a set of cues or intervening variables (e.g., essay features, domains, and rating scale usages) that are interpreted separately by experts and operational raters. Our goal in this case is to have a close correspondence between the measurement of the latent variable (i.e., writing competence) between expert and operational raters. Judgmental accuracy in Lens Model I refers to the closeness between rater's operational ratings and experts' criterion ratings of student performances including their interpretations of the cues. Wang and Engelhard (2017) applied Lens model I to evaluate rating quality in writing assessments.

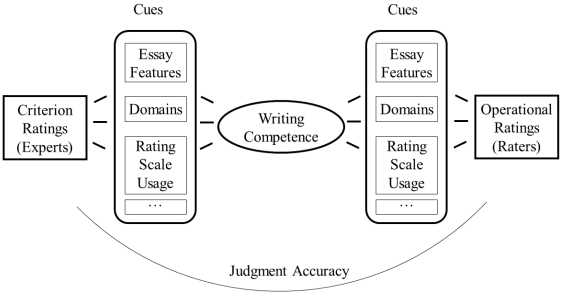


Figure 3:

Lens model I (bifocal model) for measuring writing competence

In contrast to Lens Model I, the current study focuses on a slightly different definition of a lens model. Specifically, we focus on *Lens Model II* (see Figure 4). In Lens Model II, the latent variable is rater accuracy instead of writing competence in the assessment

system. The goal is to evaluate accuracy ratings (i.e., differences between observed and criterion ratings) as responses of raters in the judgmental system. These accuracy ratings can be distinguished from the ratings modeled separately for expert and operational raters in Lens Model I.

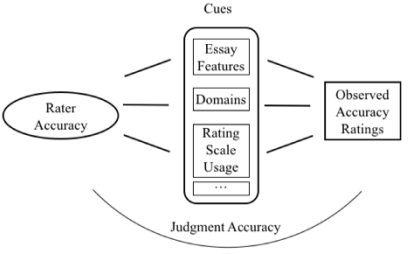


Figure 4:

Lens model II for measuring rater accuracy

As pointed out in the opening quote for this section, a defining feature of lens models is that they include two separate functions reflecting judgment and criterion systems. Brunswik (1952) primarily used correlational analyses to examine judgmental data. Multiple regression analyses are currently the most widely used method for examining data from lens-model studies of judgments (Cooksey, 1996). It is interesting to note that Hammond (1996) suggested that lens-model research may have overemphasized the role of multiple regression techniques, and that the "lens model is indifferent — a priori — to which organizing principle is employed in which task under which circumstances; it considers that to be an empirical matter" (p. 245). In our study, we suggest using psychometric models based on Rasch measurement theory and invariant measurement as an organizing principle (Engelhard, 2013). As pointed out earlier, the majority of analyses conducted with lens models are regression-based analyses. Lens Model I reflects this perspective very closely with the Rasch model substituted for multiple regression analyses.

How can we connect these two perspectives to improve rater-mediated assessments?

Accuracy ...refers to closeness of an observation to the quality intended to be observed

(Kendall & Buckland, 1957, p. 224)

Researchers have adopted several different statistical approaches for analyzing data for lens-model studies. First, the ratings have been modeled directly using correlational and multiple regression analyses (Brunswik 1952; Cooksey, 1996; Hammond, Hursch, and Todd, 1964; Hursch, Hammond, & Hursch, 1964; Tucker, 1964). Cooksey (1986) provided an informative example of using a lens model approach to examine teacher judgments of student reading achievement. In this study, student scores on standardized reading achievement tests define the ecological or criterion system with three cues (i.e., social economic status, reading ability, and oral language ability). In a similar fashion, the

judgmental system was defined based on the relationship between teacher judgments and the same set of cues. Regression-based indices were used to compare the ecological and judgmental systems. Cooksey, Freebody, and Wyatt-Smith (2007) also applied a lens model to study teacher's judgments of writing achievement. The drawback of this methodology is that each person's judgment is compared against the criterion individually; that said, separate regression analyses are required for each judge.

A second approach is to use IRT models that are developed within the scaling tradition. Researchers can obtain individual-level estimates using various IRT models in one analysis instead of separate multiple-regression analyses. For example, Engelhard (2013) proposed the use of a Many-Facet Rasch Model to examine the lens model I for measuring writing proficiency.

Finally, it is possible to model the criterion and judgmental systems as the distances between the ratings from each system. The lens model for measuring rater accuracy based on this approach can be best represented by the RAM. RAM has been proposed and applied to evaluate rater accuracy in writing assessments (Engelhard, 1996, 2013; Wolfe, Jiao, & Song, 2014). We illustrate the correspondence between the Lens Model II and the RAM. Specifically, we use the distances between the ratings of expert raters and the operational raters to define accuracy ratings which are analyzed in the judgment system of Lens Model II. RAM analyzes the accuracy ratings that are direct measures of rater accuracy.

In addition, there are several advantages of using Rasch measurement theory over regression-based approaches for judgment studies. First of all, multiple regression analyses may lead to a piecemeal approach with an array of separate analyses. Cooksey (1996) provides ample illustrations of these types of analyses within the context of judgment studies. Our approach based on Rasch measurement theory provides a coherent view for analyzing rater-mediated assessments. Second, it is hard to substantively conceptualize the focal point (i.e., object of measurement) when a regression-based approach is used. In this study, we describe two Rasch-based approaches that focus on either students or raters as the object of measurement. Our approach offers the advantages of obtaining invariant indicators of rating quality under appropriate conditions. Lastly, we would like to stress the value of Wright Maps that define an underlying continuum, and provide the opportunity to visualize and understand rater-mediated measurement as a line representing the construct or latent variable of interest.

Illustrative data analyses

In this study, we use illustrative data analyses to highlight the use of the RAM and Brunswikian lens model as a promising way to bring together psychometric and cognitive perspectives related to evaluating rater judgments. Specifically, we conducted a secondary data analysis with the use of RAM to examine differential rater functioning as one of the sources causing inaccurate ratings through the lens. The data, which were originally collected and analyzed by Wang, Engelhard, Raczynski, Song, and Wolfe (2017), were part of a statewide writing assessment program for Grade 7 students in a southeastern state of the United States.

Participants

According to Wang et al. (2017)'s data collection procedure, twenty well-trained operational raters were randomly chosen from a larger rater pool. The group of raters scored a random sample of 100 essays. This set of essays was used as training essays to evaluate rater performance prior to the actual operational scoring. The design was fully crossed with all of the raters rating all of the essays. A panel of three experts who provided the training and picked the training essays assigned the criterion ratings for these 100 essays.

Instrument

The writing assessment was document based, that is students were asked to write an essay based on a prompt. The essays were scored analytically in two domains: (a) idea development, organization, and coherence (IDOC Domain), and (b) language usage and conventions (LUC Domain). IDOC Domain was scored using a category of 0-4, and LUC domain was rated from 0-3. A higher score indicates better proficiency in a specific writing domain.

Procedures

In our study, exact matches between operational and criterion ratings from the panel of expert raters are assigned an accuracy rating of 1, while other discrepancies are assigned a 0. Higher scores reflect higher levels scoring accuracy for raters. In other words, accuracy ratings are dichotomized (0=inaccurate rating, 1=accurate ratings).

The RAM includes three facets: Raters, essays and domains. We used the Facets computer program (Linacre, 2015) to analyze the dichotomous accuracy ratings. The general RAM model can be expressed as follows:

$$\mathrm{Ln}[P_{nmik} / P_{nmik\text{-}1}] = \beta_n - \delta_m - \lambda_i - \tau_k \quad (1)$$

where

P_{nmik} = probability of rater n assigning an accurate rating to benchmark essay m for domain i,

$P_{nmik\text{-}1}$ = probability of rater n assigning an inaccurate rating to benchmark essay m for domain i,

β_n = accuracy of rater n,

δ_m = difficulty of assigning an accurate rating to benchmark essay m,

λ_i = difficulty of assigning an accurate rating for domain i, and

τ_k = difficulty of accuracy-rating category k relative to category k-1.

Next, we examine an interaction effect between rater accuracy measures and domain facet using the model as below:

$$\text{Ln}[P_{\text{nmik}} / P_{\text{nmik-1}}] = \beta_n - \delta_m - \lambda_i - \beta_n\lambda_i - \tau_k \quad (2)$$

where $\beta_n\lambda_i$ represents the interaction effect between rater and domains.

The τ_k parameter is not estimated in this study because the accuracy ratings are dichotomous. However, we included it here because it is possible to apply this model to polytomous accuracy ratings, in which case the threshold parameter would be included.

Results

Summary statistics for the calibrated facets are shown in Table 2. The Wright Map is shown in Figure 5. The reliability of separation for rater accuracy is .47, and the Chi-square test for variation among raters is statistically significant ($\chi^2 = 35.6$, $df = 19$, $p < .05$). Table 3 shows the detailed analyses of accuracy for each rater. The mean accuracy measure for raters is .63 logits with a standard deviation for .22. Rater 2702 is the most accurate rater with a measure of 1.02 logits, and Rater 2696 is the least accurate rater with an accuracy measure of .55 logits. Based on the standardized Outfit and Infit values, Rater 2569 appears to be exhibiting misfit.

Table 2:

Summary statistics for Rater Accuracy Model

	Rater	Essays	Domains
Measure			
Mean	.63	.00	.00
SD	.22	.76	.62
N	20	100	2
Infit MSE			
Mean	1.00	1.00	1.00
SD	.06	.15	.00
Outfit MSE			
Mean	1.00	1.00	1.00
SD	.10	.20	.01
Separation statistics			
Reliability of separation	.47	.77	.99
Chi-square (χ^2)	35.6*	348.4*	154.1*
df	19	99	1

Note. MSE = mean square error, * $p < .05$.

Table 3:

Accuracy measures and fit statistics for raters

Rater ID	Accuracy (Prop.)	Measure (Logits)	S.E.	Infit MSE	Infit Z	Outfit MSE	Outfit Z	Slope
2702	0.70	1.02	0.17	1.07	1.01	1.10	0.86	0.84
2744	0.69	0.91	0.16	0.98	-0.21	0.92	-0.76	1.07
3051	0.67	0.83	0.16	1.05	0.78	1.16	1.53	0.84
3271	0.67	0.83	0.16	1.07	1.10	1.08	0.76	0.82
1714	0.66	0.81	0.16	0.95	-0.82	0.92	-0.79	1.14
2505	0.65	0.73	0.16	0.99	-0.19	0.97	-0.26	1.04
3076	0.65	0.73	0.16	0.99	-0.13	0.98	-0.16	1.03
3083	0.65	0.76	0.16	1.03	0.42	1.06	0.66	0.91
3372	0.65	0.73	0.16	1.04	0.59	1.00	-0.01	0.93
698	0.64	0.70	0.16	0.90	-1.76	0.84	-1.79	1.31
3153	0.64	0.70	0.16	0.91	-1.49	0.86	-1.52	1.26
2911	0.63	0.63	0.16	0.93	-1.26	0.89	-1.20	1.23
2423	0.61	0.53	0.16	0.99	-0.15	1.04	0.54	1.00
3084	0.60	0.48	0.16	0.97	-0.57	0.93	-0.87	1.13
2020	0.59	0.44	0.15	0.96	-0.81	0.93	-0.82	1.16
2905	0.58	0.41	0.15	0.98	-0.39	0.95	-0.59	1.09
730	0.57	0.36	0.15	1.08	1.53	1.10	1.24	0.70
2481	0.57	0.36	0.15	1.02	0.37	1.03	0.43	0.92
2569	0.57	0.34	0.15	1.13	2.39*	1.23	2.85*	0.48
2696	0.55	0.25	0.15	0.98	-0.44	0.96	-0.51	1.09

Note. Accuracy is the proportion of accurate ratings. Raters are ordered based on measures (logits). SE = standard error, MSE=mean square error, and * $p < .05$.

As shown in Table 2, the benchmark essays are centered at zero with a standard deviation of .76. Overall, the benchmark essay accuracy measures have relatively good fit to the model. Measures for domain accuracy are also centered at zero. Domain IDOC has a measure of .44 logits and Domain LUC has a measure of -.44 logits (Table 4). IDOC seems to be more difficult for raters to score accurately than LUC. The reliability of separation is .99, and the differences among the domain locations on the logit scale are statistically significant ($\chi^2 = 154.1$, $df = 1$, $p < .05$)

Table 4:

Summary statistics for Rater Accuracy Model by Domain

Domains	Accuracy	Measure	SE	Infit MSE	Infit Z	Outfit MSE	Outfit Z	Slope
IDOC	0.54	0.44	0.05	1.00	-0.04	1.00	0.07	1.00
LUC	0.72	-0.44	0.05	1.00	0.02	0.99	-0.15	1.00

Note. IDOC = idea, development, organization, and cohesion, LUC = language usage and convention, SE = standard error, and MSE = mean square error.

We also included an interaction term (i.e., domain by rater facets) in the model. We used *t*-tests to compare the differences of accuracy measures between domains for each rater. Results indicate that three raters have significantly different accuracy measures between the two domains (Table 5). Specifically, Raters 3271 and 2905 appear to be significantly more accurate in scoring Domain IDOC than Domain LUC. On the contrary, Rater 3084 seems to be significantly more accurate in Domain LUC than Domain IDOC.

In order to interpret these results in terms of their substantive implications, it is informative to relate these results to the five aspects of inaccuracy described in Table 1. Specifically, *rater inaccuracy* is the tendency on the part of raters to consistently provide higher or lower ratings overall. The illustrative data in this study suggest that the individual raters vary in their levels of inaccuracy. The Wright Map (Figure 5) provides a visual display of where each rater is located on the accuracy continuum. The raters are not equivalent in terms of accuracy rates. The data also provide evidence of domain variation in inaccuracy (halo inaccuracy). Some raters appear to vary in their accuracy rates as a function of domain. Overall, there were differences in rater accuracy between the two domains, where the IDOC domain was more difficult for raters to score accurately as compared to the LUC domain.

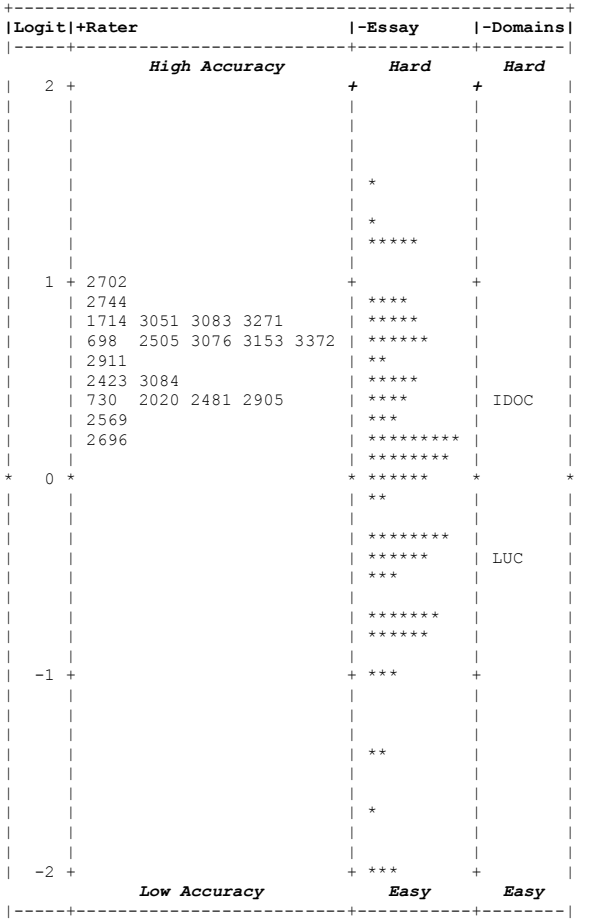
Next, *response set inaccuracy* implies that a rater interprets and uses rating scale categories in an idiosyncratic fashion. Because the accuracy data in this study are dichotomous, this issue is moot. Third, *score range inaccuracy* is observed in these data with the benchmark essays varying in difficulty to rate accurately as shown on the Wright Map (Figure 5). Further research is needed on why certain essays appear to be more accurately rated than other essays. Finally, there was evidence of an *inaccuracy interaction effect* between raters and domains. This result suggests that rater effects are not additive, and that the domain facet is not invariant across raters. In other words, the relative ordering of the domains in terms of the difficulty to assign accurate ratings was not the same for all of the raters.

Table 5:

Analysis of differential rater functioning across domains

	IDOC Domain		LUC Domain				
Rater	**Measure**	**SE**	**Measure**	**SE**	**Contrast**	**t-value**	**Prob**
3271	1.15	0.22	0.47	0.23	0.68	2.14*	0.03
2905	0.72	0.21	0.08	0.22	0.65	2.13*	0.03
2744	1.15	0.22	0.63	0.23	0.52	1.62	0.11
2423	0.68	0.21	0.37	0.22	0.31	1.01	0.31
2702	1.10	0.22	0.91	0.25	0.18	0.56	0.58
3051	0.91	0.22	0.74	0.24	0.17	0.53	0.59
3083	0.82	0.21	0.68	0.24	0.14	0.42	0.67
730	0.41	0.21	0.32	0.22	0.09	0.30	0.76
2696	0.27	0.21	0.22	0.22	0.05	0.18	0.86
2569	0.36	0.21	0.32	0.22	0.05	0.15	0.88
2481	0.36	0.21	0.37	0.22	0.00	-0.01	0.99
2505	0.72	0.21	0.74	0.24	-0.01	-0.04	0.97
3076	0.72	0.21	0.74	0.24	-0.01	-0.04	0.97
698	0.63	0.21	0.79	0.24	-0.16	-0.50	0.62
1714	0.72	0.21	0.91	0.25	-0.19	-0.58	0.56
3372	0.63	0.21	0.85	0.24	-0.22	-0.68	0.50
2911	0.45	0.21	0.85	0.24	-0.40	-1.23	0.22
3153	0.50	0.21	0.98	0.25	-0.48	-1.45	0.15
2020	0.18	0.21	0.74	0.24	-0.56	-1.73	0.08
3084	0.09	0.22	0.98	0.25	-0.89	-2.68*	0.01

Note. IDOC = ideas, development, organization, and cohesion, LUC = language usage and conventions, and SE = standard errors, * $p < .05$.



Note. IDOC = ideas, development, organization, and cohesion, LUC = language usage and conventions.

Figure 5:

Wright Map for Rater Accuracy Model

Discussion

In this study, we briefly discussed two perspectives on evaluating the quality of ratings in rater-mediated assessments: a psychometric perspective and a cognitive perspective. As shown in Figure 1, rater-mediated assessments rely on both perspectives to have reliable, valid, and fair ratings in a rater-mediated assessment system of performances. Much of the current research on rating quality has been dominated by a psychometric perspective with relatively little research on the cognitive processes of human raters. In order to meaningfully evaluate and interpret the quality of ratings, it is important to explicitly consider both

theory of measurement and theory of rater cognition. Ideally, these two perspectives should be complementary and congruent. The psychometric perspective used in this study is based on Rasch measurement theory, and the cognitive perspective is based on Brunswik's lens model. In particular, we emphasized the use of a rater accuracy model (RAM) to illustrate our major points.

Our study was guided by the following three questions:

- What psychometric perspectives can be used to evaluate ratings in rater-mediated assessments?
- What cognitive perspectives can provide guidance on how to model judgments obtained in rater-mediated assessments?
- How can we connect these two theoretical perspectives to improve rater-mediated assessments?

In answer to the first question, we believe that a scaling perspective based on item response theory in general and Rasch measurement theory in particular provides the best match to the models of judgment in rater-mediated assessments. Rasch measurement theory specifies the requirements necessary for developing and maintaining a psychometrically sound performance assessment system. There are two versions of the Rasch model that can be used to evaluate rater accuracy. A Rasch model with observed ratings and a Rasch model with accuracy ratings which is called Rater Accuracy Model. The first model focuses on two assessment systems (one based on expert raters and the second on operational raters) with the latent variable defining the object of measurement for both groups of raters. The second model (i.e., RAM) focuses on rater accuracy directly as the latent variable with the raters defined as the objects of measurement. RAM offers a direct evaluation of rater accuracy measures with accuracy ratings which are defined as the differences between observed and criterion ratings.

Turning now to the second question, we selected cognitive perspectives based on Brunswik's Lens Model as the basis for examining human judgments in rater-mediated assessments. Lens models connect the criterion system and the judgmental system which can best represent operational raters' cognition processes while making judgments. We have described two lens models. *Lens Model I* is for measuring student proficiency (e.g., writing competency) as the distal variable (Figure 3). *Lens Model II* is for measuring rater accuracy directly as the distal variable (Figure 4), which emphasizes the evaluation of the raters or judges by modeling the distances between operational ratings and criterion ratings.

The final question raises an important issue about the congruence between a statistical theory of measurement and a substantive theory regarding human cognition and judgment. Lens models can be conceptually linked to both the Many-Facet Rasch Model and the RAM with the major distinctions between the objects of measurement in two models. For both models, it is substantively useful to visualize the locations of the object of measurement on a Wright Map, to define the latent variable in terms of the specific cues used by the raters as *lens*, and to conceptualize two systems -- criterion system and judgmental system. The Many-Facet Rasch Model analyzes the two systems separately and then

compares the results. The measurement focuses on student proficiency as a latent continuum in each system, and the consistency between two systems reflects the rater accuracy. On the other hand, the RAM is used to model accuracy ratings defined as the distances between the two systems. This approach directly reflects rater accuracy by modeling it as the underlying latent trait.

Using illustrative data from a rater-mediated writing performance assessment, we demonstrated the statistical procedures for modeling rater accuracy. Specifically, we calculated accuracy ratings by matching operational ratings and the criterion ratings for individual raters. Then we used the RAM to analyze accuracy ratings to obtain the accuracy measures for individual raters, the difficulty associated with scoring accuracy for student performances (i.e., essays), and the difficulty associated with scoring accuracy for the domains that were specified in the analytic scoring rubric. To evaluate differential rater functioning, we examined the interaction between individual raters and domains. Lastly, we interpreted the statistical results of RAM based on the five potential sources of inaccuracy. These sources of inaccuracy also provide a frame of reference for interpreting the statistical results in terms of specific rater issues in operational performance assessments.

We want to stress that the statistical theories of measurement and substantive theories of human cognition and judgment for evaluating rating quality should be complementary and congruent. Ideally, research on rater-mediated assessments should balance concerns with both cognitive and psychometric perspectives. In practice, the development and evaluation of how well our theories match one another remains a challenging puzzle. As progress is made in both areas, the nexus between psychometrics and cognition for rater-mediated assessments promises to be an exciting area of research.

Finally, the title of this study reflects an indirect reference to the opening lines in *A Tale of Two Cities* (Charles Dickens, 1859):

> *It was the best of times, it was the worst of times, it was the age of wisdom, it was the age of foolishness, it was the epoch of belief, it was the epoch of incredulity, it was the season of Light, it was the season of Darkness, it was the spring of hope, it was the winter of despair…*

Some researchers who evaluate rater-mediated assessments have numerous justifiable concerns about human biases and errors (e.g., intentional and random), and their perspectives may reflect despair over the current state of the art. From our perspective, we have hope that many of the concerns about human scoring can be minimized and the promise of performance assessments become a reality in education and other contexts. In particular, we believe that explicit considerations of both psychometric and cognitive perspectives have important implications for improving the training and maintaining the quality of ratings obtained from human raters in performance assessments.

References

Andrich, D. (1978). A rating formulation for ordered response categories. *Psychometrika, 43*, 561-73.

Athanasou, J.A., & Kaufmann, E. (2015). Probability of responding: A return to the original Brunswik. *Psychological Thought, 8*(1), 7–16.

Barsalou, L. W. (1992). *Cognitive psychology: An overview for cognitive scientists*. Psychology Press.

Bergin, C., Wind, S. A., Grajeda, S., & Tsai, C.-L. (2017). Teacher evaluation: Are principals' classroom observations accurate at the conclusion of training? *Studies in Educational Evaluation*, *55*, 19–26.

Brunswik, E. (1952). *The conceptual framework of psychology*. Chicago, IL: University of Chicago Press.

Brunswik, E. (1955a). Representative design and probabilistic theory in a functional psychology. *Psychological Review, 62*(3), 193-217.

Brunswik, E. (1955b). In defense of probabilistic functionalism: A reply. *Psychological Review, 62*(3), 236-242.

Brunswik, E. (1956). *Perception and the representative design of psychological experiments* (2nd ed.). Berkeley, CA: University of California Press.

Casabianca, J. M., Junker, B. W., & Patz, R. J. (2016). Hierarchical rater models. In W. J. van der Linden (Ed.), *Handbook of item response theory* (Vol. 1, pp. 449–465). Boca Raton, FL: Chapman & Hall/CRC.

Cooksey, R. W. (1996). *Judgment analysis: Theory, methods, and applications*. United Kingdom: Emerald Group Publishing Limited.

Cooksey, R. W., Freebody, P., & Davidson, G. R. (1986). Teachers' predictions of children's early reading achievement: An application of social judgment theory. *American Educational Research Journal, 23(1),* 41-64.

Cooksey, R. W., Freebody, P., & Wyatt-Smith, C. (2007). Assessment as judgment-in-context: Analysing how teachers evaluate students' writing. *Educational Research and Evaluation, 13(5),* 401-434.

Cronbach, L. J., Gleser, G. C., Nanda, H., & Rajaratnam, N. (1972). *The dependability of behavioral measurements: Theory of generalizability for scores and profiles.* New York: Wiley.

Dickens, C. J. H. (1859). *A tale of two cities (Vol. 1).* Chapman and Hall.

Eckes, T. (2005). Examining rater effects in TestDaF writing and speaking performance assessments: A many-facet Rasch analysis. *Language Assessment Quarterly*, *2*(3), 197–221.

Eckes, T. (2015). *Introduction to many-facet Rasch measurement: Analyzing and evaluating rater mediated assessments* (2nd ed.). Frankfurt am Main: Peter Lang.

Engelhard Jr, G. (1992). The measurement of writing ability with a many-faceted Rasch model. *Applied Measurement in Education, 5(3),* 171-191.

Engelhard, G. (1994). Examining rater errors in the assessment of written composition with a many-faceted Rasch model. *Journal of Educational Measurement, 31(2),* 93-112.

Engelhard, G. (1996). Evaluating rater accuracy in performance assessments. *Journal of Educational Measurement*, *33*(1), 56-70.

Engelhard, G. (2002). Monitoring raters in performance assessments. In G. Tindal and T. Haladyna (Eds.), *Large-scale assessment programs for ALL students: Development, implementation, and analysis,* (pp. 261-287). Mahwah, NJ: Erlbaum.

Engelhard, G. (2013). *Invariant measurement: Using Rasch models in the social, behavioral, and health sciences*. New York: Routledge.

Engelhard, G., & Myford, C. (2010). Comparison of single and double assessor scoring designs for the assessment of accomplished teaching. In Garner, M., Engelhard, G., Wilson, M., & Fisher, W. (Eds.). *Advances in Rasch measurement* (Vol. 1, pp. 342-368). Maple Grove, MN: JAM Press.

Engelhard, G., & Wind, S.A. (2013). *Rating quality studies using Rasch measurement theory.* College Board Research Report 2013-3.

Guilford, J. P. (1936). *Psychometric methods.* New York: McGraw Hill.

Hammond, K. R. (1955). Probabilistic functioning and the clinical method. *Psychological review, 62(4),* 255.

Hammond, K. R. (1996). Upon reflection. *Thinking & Reasoning*, *2*(2-3), 239-248.

Hammond, K. R., Hursch, C. J., & Todd, F. J. (1964). Analyzing the components of clinical inference. *Psychological review, 71(6),* 438.

Hursch, C. J., Hammond, K. R., & Hursch, J. L. (1964). Some methodological considerations in multiple-cue probability studies. *Psychological review, 71(1),* 42.

Johnson, R. L., Penny, J. A., & Gordon, B. (2009). *Assessing performance: Designing, scoring, and validating performance tasks*. New York: Guilford Press.

Karelaia, N., & Hogarth, R.M. (2008). Determinants of linear judgment: A meta-analysis of lens model studies. *Psychological Bulletin, 134*(3), 404–426.

Kaufmann, E., Reips, U. D., & Wittmann, W. W. (2013). A critical meta-analysis of lens model studies in human judgment and decision-making. *PloS one*, *8*(12), e83528.

Kendall, M. G., & Buckland, W. R. (1957). *Dictionary of statistical terms*. Edinburgh, Scotland: Oliver and Boyd.

Lane, S. (2016). Performance assessment and accountability: Then and now. In C. Wells & M. Faulkner-Bond (Eds). *Educational measurement: From foundations to future* (pp. 356-372). New York: Guilford.

Leighton, J. P. (2012). Editorial. *Educational Measurement: Issues & Practice*, *31*(3), 48-49.

Linacre, J. M. (1989). *Many-facet Rasch measurement*. Chicago, IL: MESA Press.

Linacre, J. M. (2015) *Facets computer program for many-facet Rasch measurement, version 3.71.4.* Beaverton, Oregon: Winsteps.com

Masters, G. N. (1982). A Rasch model for partial credit scoring. *Psychometrika*, *47*(2), 149-174.

Muraki, E. (1992). A generalized partial credit model: Application of an EM algorithm. *Applied Psychological Measurement, 16*, 159–176.

Myford, C. M. (2012). Rater cognition research: Some possible directions for the future. *Educational Measurement: Issues & Practice*, *31*(3), 48-49.

Nering, M.L., & Ostini, R. (2010). *Handbook of polytomous item response theory models.* New York: Routledge.

Patterson, B.F., Wind, S.A., & Engelhard, G. (2017). Incorporating criterion ratings into model-based rater monitoring procedures using latent class signal detection theory. *Applied Psychological Measurement*, 1-20.

Postman, L., & Tolman, E. C. (1959). Brunswik's probabilistic functionalism. *Psychology: A study of a science*, *1*, 502-564.

Rasch (1960/1980). *Probabilistic models for some intelligence and attainment tests.* Copenhagen: Danish Institute for Educational Research. (Expanded edition, Chicago: University of Chicago Press, 1980).

Samejima, F. (1969). Estimation of latent ability using a response pattern of graded scores. *Psychometrika Monograph*, No. 17.

Tucker, L. R. (1964). A suggested alternative formulation in the developments by Hursch, Hammond, and Hursch, and by Hammond, Hursch, and Todd. *Psychological Review, 71*(6), 528-530.

von Eye, A., & Mun E. Y. (2005). *Analyzing rater agreement: Manifest variable methods*. Mahwah, NJ: Erlbaum.

Wang, J., Engelhard, G., & Wolfe, E. W. (2016). Evaluating rater accuracy in rater-mediated assessments with an unfolding model. *Educational and Psychological Measurement, 76*, 1005–1025.

Wang, J., Engelhard, G., Raczynski, K., Song, T., & Wolfe, E. W. (2017). Evaluating rater accuracy and perception for integrated writing assessments using a mixed-methods approach. *Assessing Writing, 33*, 36-47.

Wesolowski, B. W., & Wind, S. A. (in press). Investigating rater accuracy in the context of secondary-level solo instrumental music. *Musicae Scientae*.

Wesolowski, B., Wind, S.A., & Engelhard, G. (2016). Examining rater precision in music performance assessment: An analysis of rating scale structure using the multifaceted Rasch partial credit model. *Music Perception, 33*(5), 662–678.

Wind, S.A., & Engelhard, G. (2013). How invariant and accurate are domain ratings in writing assessment? *Assessing Writing, 18,* 278–299.

Wolfe, E. W. (2014). *Methods for monitoring rating quality: Current practices and suggested changes*. Iowa City, IA: Pearson.

Wolfe, E. W., Jiao, H., & Song, T. (2014). A family of rater accuracy models. *Journal of Applied Measurement*, *16(2),* 153-160.

Wolfe, E. W., & McVay, A. (2012). Application of latent trait models to identifying substantively interesting raters. *Educational Measurement: Issues and Practice*, *31*(3), 31–37.

Wang, J. & Engelhard, G. (2017). Using a multifocal lens model and Rasch measurement theory to evaluate rating quality in writing assessments. Pensamiento Educativo: *Journal of Latin-American Educational Research, 54(2)*, 1-16.

Wind, S. A., & Peterson, M. E. (2017). A systematic review of methods for evaluating rating quality in language assessment. *Language Testing*, doi: 10.1177/0265532216686999

Psychological Test and Assessment Modeling, Volume 60, 2018 (1), 53-80

Modeling rater effects using a combination of Generalizability Theory and IRT

Jinnie Choi[1] *& Mark R. Wilson*[2]

Abstract

Motivated by papers on approaches to combine generalizability theory (GT) and item response theory (IRT), we suggest an approach that extends previous research to more complex measurement situations, such as those with multiple human raters. The proposed model is a logistic mixed model that contains the variance components needed for the multivariate generalizability coefficients. Once properly set-up, we can estimate the model by straightforward maximum likelihood estimation. We illustrate the use of the proposed method with a real multidimensional polytomous item response data set from classroom assessment that involved multiple human raters in scoring.

Keywords: generalizability theory, item response theory, rater effect, generalized linear mixed model

[1]*Correspondence concerning this article should be addressed to:* Jinnie Choi, Research Scientist at Pearson, 221 River Street, Hoboken, NJ 07030; email: jinnie.choi@pearson.com.

[2]University of California, Berkeley

While item response theory (IRT; Lord, 1980; Rasch, 1960) and generalizability theory (GT; Brennan, 2001; Cronbach, Gleser, Nanda, & Rajaratnam, 1972) share common goals in educational and psychological research in order to provide evidence of the quality of measurement, IRT and GT have evolved into two separate domains of knowledge and practice in psychometrics that rarely communicate with one another. In practice, it is often recommended that researchers and practitioners be able to use and understand both methods, and to distinguish the same term with different meanings (e.g., reliability) or different terms with similar meanings (e.g., unidimensional testlet design in IRT and p x (i : h) design in GT), neither of which is desirable or practical. The separate foundations and development of these two techniques have resulted in a wide gap between the two approaches and have hampered collaboration between those who specialize in each. Additionally, despite the theories' extensive applicability, IRT and GT are often applied to somewhat different areas of research and practice. For example, applications of GT are often found in studies on reliability and sampling variability of smaller-scale assessments. Meanwhile, IRT is, relatively speaking, more commonly and more widely employed, than GT for developing large-scale educational assessments, such as the Programme for International Student Assessment (PISA) and the ones currently used by the US National Center for Education Statistics (NCES), and the products of large testing companies such as Educational Testing Service (ETS). Moreover, most advanced applications of IRT and GT take only one approach, not both. Considering the advantages of the two theories, this limitation and bias in usage call for an alternative approach to promote a more efficient and unified way to deliver the information that can be provided by IRT and GT together.

Several researchers have undertaken efforts to find the solution to this separation. For example, the researchers either: (a) highlight the differences but suggest using both, consecutively (Linacre, 1993), (b) discuss the link between the models (Kolen & Harris, 1987; Patz, Junker, Johnson, & Mariano, 2002), or (c) propose a new approach to combine the two (Briggs & Wilson, 2007).

Linacre (1993) emphasized the difference between IRT and GT and suggested that decision-makers select either one or the other, or use both, based on the purpose of the analysis. Many researchers took this advice and used both the IRT and the GT models, for example, for performance assessments of English as Second Language students (Lynch & McNamara, 1998), for English assessment (MacMillan, 2000), for writing assessments of college sophomores (Sudweeks, Reeve, & Bradshaw, 2005), for problem-solving assessments (Smith & Kulikowich, 2004), and for clinical examinations (Iramaneerat, Yudkowsky, Myford, & Downing, 2008).

While Linacre's suggestion promoted the idea of combining the use of the models, the statistical notion of links between IRT and GT began to emerge when Kolen and Harris (1987) proposed a multivariate model based on a combination of IRT and GT. The model assumed that the true score in GT could be approximated by the proficiency estimate in IRT. Patz, Junker, Johnson, & Mariano (2002) proposed a new model that combines IRT and GT, namely, the hierarchical rater model (HRM), which they see as a standard generalizability theory model for rating data, with IRT distributions replacing the normal theory true score distributions that are usually implicit in inferential applications of the

model. The proposed use of the model is open to other possible extensions, although it is currently conceptualized as being used for estimation of rater effects.

These efforts motivated a noteworthy advance in combining IRT and GT, namely, the Generalizability in Item Response Modeling (GIRM) approach by Briggs & Wilson (2007), and its extensions by Choi, Briggs, & Wilson (2009) and Chien (2008). The GIRM approach provides a method for estimating traditional IRT parameters, the GT-comparable variance components, and the generalizability coefficients, not with observed scores but with the "expected item response matrix" — EIRM. By estimating a crossed random effects IRT model within a Bayesian framework, the GIRM procedure constructs the EIRM upon which GT-comparable analysis can be conducted. The steps can be described as follows:

Step 1. The probability of a correct answer is modeled using a crossed random effects item response model that considers both person and item as random variables. The model parameters are estimated using the Markov chain Monte Carlo (MCMC) method with the Gibbs sampler.

Step 2. Using the estimates from Step 1, the probability of the correct answer for each examinee answering each item is predicted to build the EIRM.

Step 3. The variance components and generalizability coefficients are estimated based on the EIRM.

Estimation of the variance components and generalizability coefficients utilizes an approach described by Kolen and Harris (1987) that calculates marginal integrals for facet effects, interaction effect, and unexplained error using the prior distributions and the predicted probabilities of IRT model parameters.

The main findings of the Briggs & Wilson (2007) study were as follows:

- GIRM estimates are comparable to GT estimates in the simple *p x i* test design where there are person and item facets alone, and with binary data.
- GIRM easily deals with the missing data problem, a problem for earlier approaches, by using the expected response matrix.
- Because GIRM combines the output from IRT with the output from GT, GIRM provides more information than either approach in isolation.
- Although GIRM adds the IRT assumptions and distributional assumptions to the GT sampling assumptions, GIRM is robust to misspecification of item response function and prior distributions.

In the multidimensional extension of the same method, Choi, Briggs, and Wilson (2009) found that the difference between GIRM and traditional GT estimates is more noticeable, with GIRM producing more stable variance component estimates and generalizability coefficients than traditional GT. Noticeable patterns of differences included the following:

- GIRM item variance estimates were smaller and more stable than GT,

- GIRM error variance (*pi* + *e*) estimates were larger and more stable than GT residual error variance (*pie*) estimates, and
- GIRM generalizability coefficients were generally larger and more precise than GT generalizability coefficients.

With the testlet extension of the procedure by Chien (2008), (a) the estimates of the person, the testlet, the interaction between the item and testlet, and the residual error variance estimates were found to be comparable to traditional GT estimates when data are generated from IRT models. (b) For the dataset generated from GT models, the interaction and residual variance estimates were slightly larger while person variance estimates were slightly smaller than traditional GT estimates. (c) The person-testlet interaction variance estimates were slightly larger than the traditional GT estimates for all conditions. (d) When the sample size was small, the discrepancy between the estimated universe mean scores in GT and the expected data in GIRM increased. (e) MCMC standard errors were notably underestimated for all variance components.

The mixed results from the studies of GIRM and its extensions yielded interesting questions.

- What is the statistical nature of the EIRM? The main advantage of the GIRM procedure comes from this matrix, coupled with the MCMC estimation within a Bayesian framework. This is a notable departure from the analogous-ANOVA estimation of traditional GT that brings the following benefits: (a) the variance component estimates are non-negative and (b) the problems that arise from unbalanced designs and missing data are easily taken care of. However, the extension studies revealed that the EIRM does not theoretically guarantee the equivalence of GIRM and traditional GT estimates in more complicated test conditions.
- Then, what is the benefit of having the extra step that requires multiple sets of assumptions and true parameters for each stage?
- Are there other ways to deal with the negative variance estimate problem in traditional GT and the missing data problem, and still get comparable results?
- Among the different approaches, which procedure gives more correct estimates?

These questions led to the search for an alternative strategy that requires simple one-stage modeling, and possibly non-Bayesian estimation that produces GT-comparable results, while capturing the essence of having random person and item parameters and variance components. The following section describes a different approach, one within the GLLAMM framework, to combine GT and IRT. In the next section, we explain how the random person and item parameters are estimated using a Laplace approximation implemented in the lmer() function (Bates, Maechler, Bolker, & Walker, 2014) in the R Statistical Environment (R Development Core Team, 2017). After that, we demonstrate applications of our approach to classroom assessment data from the 2008-2009 Carbon Cycle project, which includes 1,371 students' responses to 19 items, rated by 8 raters.

The proposed model

This paper uses a generalized linear latent and mixed model (GLLAMM; Skrondal & Rabe-Hesketh, 2004; Rabe-Hesketh, Skrondal, & Pickles, 2004) approach as an alternative to existing efforts to combine GT and IRT. GLLAMM offers a flexible one-stage modeling framework for a combination of crossed random effects IRT models and GT variance components models. The model is relatively straight-forward to formulate and easily expandable to more complex measurement situations such as multidimensionality, polytomous data, and multiple raters. In this section, we describe how the model specifies a latent threshold parameter as a function of cross-classified person, item, and rater random effects and the variance components for each facet.

GLLAMM is an extended family of generalized linear mixed models (Breslow & Clayton, 1993; Fahrmeir & Tutz, 2001), which was developed in the spirit of synthesizing a wide variety of latent variable models used in different academic disciplines. This general model framework has three parts. The response model formulates the relationship between the latent variables and the observed responses via the linear predictor and link function, which accommodates various kinds of response types. The structural model specifies the relationship between the latent variables at several levels. Finally, the distribution of disturbances for the latent variables is specified. For more details, see Rabe-Hesketh et al. (2004) and Skrondal & Rabe-Hesketh (2004). In this section, GT and IRT are introduced as special case of GLLAMM. Then, the GLLAMM approach to combining GT and IRT is detailed.

GT in the GLLAMM framework

The GLLAMM framework for traditional GT models consists of the response model for continuous responses, and multiple levels of crossing between latent variables. A multifaceted measurement design with person, item and rater facets will be used for an example. First, suppose, for the moment, that there is a continuous observed score for person *j* on item *i* rated by rater *k* which is modeled as

$$y_{ijk} = \nu_{ijk} + \epsilon_{ijk}, \tag{1}$$

Where the error ϵ_{ijk} has variance σ and the linear predictor ν_{ijk} is defined as a three-way random effects model

$$\nu_{ijk} = \beta_0 + \eta_{1i}^{(2)} + \eta_{2j}^{(2)} + \eta_{3k}^{(2)}, \tag{2}$$

where β_0 is the grand mean in the universe of admissible observations. $\eta_{1i}^{(2)}$, $\eta_{2j}^{(2)}$, and $\eta_{3k}^{(2)}$ are interpreted as item, person, and rater effects, respectively. The (2) superscript denotes that the units of the variable vary at level 2. The subscript starts with a number identifier for latent variables and the alphabetical identifier for units. These effects are not considered nested but crossed because each person could have answered any item,

each person's responses could have been rated by any rater, and each rater could have rated any item. The model with interactions between the random effects can be written as a reduced form multilevel model,

$$\nu_{ijk} = \beta_0 + \eta_{1i}^{(3)} + \eta_{2j}^{(3)} + \eta_{3k}^{(3)} + \eta_{4ij}^{(2)} + \eta_{5jk}^{(2)} + \eta_{6ik}^{(2)}, \tag{3}$$

assuming that the interaction effects are latent variables varying at level 2 and the cluster-specific main effects are varying at level 3.

In traditional GT, the latent variables are assumed to equal the disturbances without covariates or factor loadings. Thus, they are described as the random intercepts such that $\eta_{1i}^{(3)} = \zeta_{1i}^{(3)}$, $\eta_{2j}^{(3)} = \zeta_{2j}^{(3)}$, $\eta_{3k}^{(3)} = \zeta_{3k}^{(3)}$, $\eta_{1ij}^{(2)} = \zeta_{1ij}^{(2)}$, $\eta_{2jk}^{(2)} = \zeta_{2jk}^{(2)}$, and $\eta_{3ik}^{(2)} = \zeta_{3ik}^{(2)}$. The distribution of the disturbances can be specified as $\zeta_{1i}^{(3)} \sim N(0, \psi_1^{(3)})$, $\zeta_{2j}^{(3)} \sim N(0, \psi_2^{(3)})$, $\zeta_{3k}^{(3)} \sim N(0, \psi_3^{(3)})$, $\zeta_{4ij}^{(2)} \sim N(0, \psi_4^{(2)})$, $\zeta_{5jk}^{(2)} \sim N(0, \psi_5^{(2)})$, and $\zeta_{6ik}^{(2)} \sim N(0, \psi_6^{(2)})$.

The generalizability coefficient is defined as the ratio of the universe score variance to the sum of the universe score variance and relative error variance.

$$E(\hat{\rho}_J^2) = \frac{\widehat{Var}\left(\eta_{2j}^{(3)}\right)}{\widehat{Var}\left(\eta_{2j}^{(3)}\right) + \widehat{Var}\left(\eta_{4ij}^{(2)}\right) + \widehat{Var}\left(\eta_{5jk}^{(2)}\right) + \widehat{Var}(\epsilon_{ijk})} = \frac{\hat{\varphi}_2^{(3)}}{\hat{\varphi}_2^{(3)} + \hat{\varphi}_4^{(2)} + \hat{\varphi}_5^{(2)} + \hat{\sigma}} \tag{4}$$

The index of dependability is defined as the ratio of the universe score variance to the total variance that includes the universe score variance and absolute error variance.

$$\begin{aligned} \widehat{\Phi_J} &= \frac{\widehat{Var}\left(\eta_{2j}^{(3)}\right)}{\widehat{Var}\left(\eta_{1i}^{(3)}\right) + \widehat{Var}\left(\eta_{2j}^{(3)}\right) + \widehat{Var}\left(\eta_{3k}^{(3)}\right) + \widehat{Var}\left(\eta_{4ij}^{(2)}\right) + \widehat{Var}\left(\eta_{5jk}^{(2)}\right) + \widehat{Var}\left(\eta_{6ik}^{(2)}\right) + \widehat{Var}(\epsilon_{ijk})} \\ &= \frac{\hat{\varphi}_2^{(3)}}{\hat{\varphi}_1^{(3)} + \hat{\varphi}_2^{(3)} + \hat{\varphi}_3^{(3)} + \hat{\varphi}_4^{(2)} + \hat{\varphi}_5^{(2)} + \hat{\varphi}_6^{(2)} + \hat{\sigma}} \end{aligned} \tag{5}$$

IRT in the GLLAMM framework

The GLLAMM framework for traditional IRT models requires a response model for categorical responses, two levels of nesting, and a latent variable for persons (Skrondal & Rabe-Hesketh, 2004). An important difference between IRT and GT is the type of response that is modeled. As item responses are categorical, a classical latent response model can be formulated as introduced by Pearson (1901). The underlying continuous response y_{ij}^* is modeled[3] as

$$y_{ij}^* = \nu_{ij} + \epsilon_{ij}. \tag{6}$$

[3] *Note* that for continuous responses such as the ones modeled in traditional GT, $y_{ij}^* = y_{ij}$.

ν_{ij} is the log odds of correct answers to items *i* for person *j* conditional on person ability η_j and ϵ_{ij} has a logistic distribution, ϵ_{ij}~logistic, that has mean 0 and variance $\frac{\pi^2}{3}$. This is the same as writing the model with a logit link function, $\text{logit}\big(P(y_{ij}=1|\eta_j)\big)=\nu_{ij}$ for dichotomous responses. Other distributions such as probit are used in certain cases when it is more appropriate to assume 1 for the error variance of the latent variable and when it is not desired to interpret the coefficients in terms of odds ratios.

For dichotomous responses, the observed response y_{ij} is defined as $y_{ij}=1$ if $y_{ij}^*>0$, and $y_{ij}=0$ otherwise.

$$\ln\left(\frac{\Pr(y_{ij}^*>0|\eta_j)}{\Pr(y_{ij}^*\leq 0|\eta_j)}\right)=\nu_{ij} \tag{7}$$

The Rasch model (Rasch, 1960) or the one-parameter (1PL) model, has a random intercept for persons and a fixed parameter for items denoted by

$$\nu_{ij}=\eta_j-\beta_i \tag{8}$$

where η_j is the latent variable for person *j*, η_j~$N(0,1)$, and β_i is the fixed effect for item *i*. In the two-parameter logistic model, or 2PL model, a slope parameter or a factor loading is added for each item such that

$$\nu_{ij}=\lambda_i(\eta_j-\beta_i) \tag{9}$$

where λ_i represents item discrimination.

For polytomous items, let *C* be the number of categories for an item. Assume that the category score is defined as *c* = 1, …, *C*-1, also representing the steps between the scores. In the polytomous case, each category score y_{icj} for the category score c is modeled with a separate linear predictor ν_{icj}. Depending on data and intended interpretation, one can specify the model differently. For example, using the sequential stage continuation ratio logit scheme, y_{icj} takes the value of 1 if $y_{icj}^*>c$ and 0 if $y_{icj}^*=c$ for category *c*. Then

$$\ln\left(\frac{\Pr(y_{icj}^*>c|\eta_j)}{\Pr(y_{icj}^*=c|\eta_j)}\right)=\nu_{icj}. \tag{10}$$

Using the adjacent category logit scheme (Agresti & Kateri, 2011), y_{icj} takes the value of 1 if $y_{icj}^*=c$ and 0 if $y_{icj}^*=c-1$ for category *c*. The adjacent category logit specification is widely used in polytomous item response models such as the rating scale model (Andrich, 1978) and the partial credit model (Masters, 1982):

$$\ln\left(\frac{\Pr(y_{icj}^*=c|\eta_j)}{\Pr(y_{icj}^*=c-1|\eta_j)}\right)=\nu_{icj}. \tag{11}$$

In cumulative models for ordered categories, y_{icj} takes the value of 1 if $y^*_{icj} > c$ and 0 if $y^*_{icj} \leq c$ for category c. The graded response model (Samejima, 1969) is specified using cumulative probabilities and threshold parameters. When a logit link is used, the model is specified as,

$$\ln\left(\frac{\Pr(y^*_{icj}>c|\eta_j)}{\Pr(y^*_{icj}\leq c|\eta_j)}\right) = \nu_{icj}. \tag{12}$$

In the case of the partial credit model, the linear predictor is specified as

$$\nu_{icj} = c\eta_j - \beta_{ic} \tag{13}$$

with β_{ic} representing the c^{th} step difficulty for item i. The graded response model uses a set of ordered threshold parameters κ_c such that

$$\nu_{icj} = \kappa_c\eta_j - \beta_{ic} \tag{14}$$

where κ_c can be viewed as the factor loadings for each step.

Zheng & Rabe-Hesketh (2007) presented the Rasch, 2PL, partial credit model and rating scale model using the additional parameters for covariates for latent variables and other parameters, so that the structure of item loading and scoring is more explicit.

Theoretical link and justification of combining GT and IRT using GLLAMM

The combination of GT and IRT ideas become simpler when GT and IRT features are expressed in the same GLLAMM language. The key elements of the combined model include: the latent response specification, a logit link, and a linear predictor specified as a crossed random effects model.

For dichotomous responses, the underlying continuous response to the i^{th} item of the j^{th} person rated by the k^{th} rater is modeled using a classical latent response model:

$$y^*_{ijk} = \nu_{ijk} + \epsilon_{ijk}, \quad \epsilon_{ijk} \sim logistic(\mu, \frac{\pi^2}{3}), \tag{15}$$

where ν_{ijk} designates the true score for every possible pair of units i, j, and k, or the expected responses. The observed response y_{ijk} is modeled as a threshold that takes the value of 1 if $y^*_{ijk} > 0$ and 0 otherwise.

The linear predictor ν_{ijk} is defined as a crossed random effects model with or without interaction. For simplicity, a model without interaction is presented here as

$$\nu_{ijk} = \beta_0 + \eta^{(2)}_{1i} + \eta^{(2)}_{2j} + \eta^{(2)}_{3k} \tag{16}$$

where β_0 is the average logit of the probability of response 1 averaging over all persons, items, and raters. Note that the effects for persons and items are not considered nested but crossed because each person could have answered each item. As above, the (2) superscript denotes that the units of the variable vary at level 2. $\eta_{1i}^{(2)}$ is the first latent variable that varies among items (i = 1,…,I) at level 2, and $\eta_{2j}^{(2)}$ is the second latent variable at level 2 that varies among persons (j = 1, …, J). $\eta_{3k}^{(2)}$ is the third latent variable at level 2 that varies among raters (k = 1, …, K). The interpretations of $\eta_{1i}^{(2)}$, $\eta_{2j}^{(2)}$, and $\eta_{3k}^{(2)}$ are item easiness, person ability, and rater leniency, respectively. If the addition signs are switched to subtraction signs for $\eta_{1i}^{(2)}$ and $\eta_{3k}^{(2)}$, the interpretations are also reversed as item difficulty and rater severity.

The latent variables are assumed to equal the disturbances, $\eta_{1i}^{(2)} = \zeta_{1i}^{(2)}$, $\eta_{2j}^{(2)} = \zeta_{2j}^{(2)}$ and $\eta_{3k}^{(2)} = \zeta_{3k}^{(2)}$, which are specified as $\zeta_{1i}^{(2)} \sim N(0, \psi_1^{(2)})$, $\zeta_{2j}^{(2)} \sim N(0, \psi_2^{(2)})$, and $\zeta_{3k}^{(2)} \sim N(0, \psi_3^{(2)})$, corresponding to the assumptions of traditional GT.

In the case of person-specific unobserved heterogeneity, the model is specified as

$$\nu_{ijk} = \beta_0 + \eta_{1i}^{(2)} + \sum_{d=1}^{D} \lambda_{id}\eta_{2jd}^{(2)} + \eta_{3k}^{(2)}, \quad \zeta_{2jd}^{(2)} \sim MVN(\mathbf{0}, \mathbf{\Psi}_2^{(2)}). \tag{17}$$

with the number of dimensions D, the item factor loadings λ_{id}, and a covariance matrix $\mathbf{\Psi}$. For the Rasch model, λ_{id} is 1 if the i^{th} item maps onto the d^{th} dimension, 0 otherwise.

The continuation ratio approach is used for polytomous data, following Tutz's (1990) parameterization in his sequential stage modeling (De Boeck, Bakker, Zwitser, Nivard, Hofman, Tuerlinckx, & Partchev, 2011). y_{icjk} takes the value of 1 if $y_{icjk}^{*} > c$ and 0 if $y_{icjk}^{*} = c$, where c (c = 1, …, C–1) denotes the category score and C denotes the number of score categories including the score 0. The linear predictors ν_{icjk} for unidimensional and multidimensional cases are specified as

$$\nu_{icjk} = \beta_0 + \eta_{1ic}^{(2)} + \eta_{2j}^{(2)} + \eta_{3k}^{(2)}, \tag{18}$$

$$\nu_{icjdk} = \beta_0 + \eta_{1ic}^{(2)} + \sum_{d=1}^{D} \lambda_{id}\eta_{2jd}^{(2)} + \eta_{3k}^{(2)}, \quad \zeta_{2jd}^{(2)} \sim MVN(\mathbf{0}, \mathbf{\Psi}_2^{(2)}). \tag{19}$$

Using the variance components estimates, the generalizability coefficient $E(\hat{\rho}_j^2)$ for the person estimates is calculated. In GT terms, $E(\hat{\rho}_j^2)$ is the ratio of the universe score variance to the sum of itself plus the relative error variance. The universe score variance is defined as the variance of all the scores in the population of all the persons, items, and raters. The relative error variance means the measurement error variance relevant to the relative rank order between persons. The variance of ϵ_{ijk} is included in the denominator to take into account the variance of the underlying logit. Additionally, using the variance

component for items and raters in the model, we calculate the generalizability coefficient for measurement of item easiness, $E(\hat{\rho}_I^2)$, and rater leniency, $E(\hat{\rho}_K^2)$, in the same manner:

$$E(\hat{\rho}_J^2) = \frac{\widehat{Var}(\eta_{2j}^{(2)})}{\widehat{Var}(\eta_{2j}^{(2)}) + \widehat{Var}(\epsilon_{ijk})} = \frac{\hat{\varphi}_2^{(2)}}{\hat{\varphi}_2^{(2)} + \frac{\pi^2}{3}} \tag{20}$$

$$E(\hat{\rho}_I^2) = \frac{\widehat{Var}(\eta_{1i}^{(2)})}{\widehat{Var}(\eta_{1i}^{(2)}) + \widehat{Var}(\epsilon_{ijk})} = \frac{\hat{\varphi}_1^{(2)}}{\hat{\varphi}_1^{(2)} + \frac{\pi^2}{3}} \tag{21}$$

$$E(\hat{\rho}_K^2) = \frac{\widehat{Var}(\eta_{3k}^{(2)})}{\widehat{Var}(\eta_{3k}^{(2)}) + \widehat{Var}(\epsilon_{ijk})} = \frac{\hat{\varphi}_3^{(2)}}{\hat{\varphi}_3^{(2)} + \frac{\pi^2}{3}} \tag{22}$$

The index of dependability $\hat{\Phi}$ is the ratio of the universe score variance to the sum of itself plus the absolute error variance. Absolute error variance focuses on the measurement error variance of a person that is attributed by the measurement facets regardless of how other people do on the test. Thus, $\hat{\Phi}$ accounts for the variance related to another random facet, for example, items. The denominator also includes the variance of the underlying logit. The same logic can be extended to calculation of the indices of dependability for item easiness and rater leniency:

$$\widehat{\Phi_J} = \frac{\hat{\varphi}_2^{(2)}}{\hat{\varphi}_1^{(2)} + \hat{\varphi}_2^{(2)} + \hat{\varphi}_3^{(2)} + \frac{\pi^2}{3}} \tag{23}$$

$$\widehat{\Phi_I} = \frac{\hat{\varphi}_1^{(2)}}{\hat{\varphi}_1^{(2)} + \hat{\varphi}_2^{(2)} + \hat{\varphi}_3^{(2)} + \frac{\pi^2}{3}} \tag{24}$$

$$\widehat{\Phi_K} = \frac{\hat{\varphi}_3^{(2)}}{\hat{\varphi}_1^{(2)} + \hat{\varphi}_2^{(2)} + \hat{\varphi}_3^{(2)} + \frac{\pi^2}{3}} \tag{25}$$

In the multidimensional and/or polytomous case, we use the dimension- and category-specific variance component estimates along with the number of items in each dimension and with the number of persons who got each category score as weights to calculate the composite generalizability coefficient and the index of dependability (Brennan, 2001; Choi, Briggs & Wilson, 2009).

$$E(\hat{\rho}_d^2) = \frac{\hat{\varphi}_d^{(2)}}{\hat{\varphi}_d^{(2)} + \frac{\pi^2}{3}} \tag{26}$$

$$E(\hat{\rho}_c^2) = \frac{\hat{\varphi}_c^{(2)}}{\hat{\varphi}_c^{(2)} + \frac{\pi^2}{3}} \tag{27}$$

$$\widehat{\Phi_d} = \frac{\hat{\varphi}_d^{(2)}}{\hat{\varphi}_c^{(2)} + \hat{\varphi}_d^{(2)} + \hat{\varphi}_3^{(2)} + \frac{\pi^2}{3}} \tag{28}$$

$$\widehat{\Phi_c} = \frac{\hat{\varphi}_c^{(2)}}{\hat{\varphi}_c^{(2)} + \hat{\varphi}_d^{(2)} + \hat{\varphi}_3^{(2)} + \frac{\pi^2}{3}} \tag{29}$$

where $\hat{\varphi}_d^{(2)}$ is the composite of universe score variance on the person side, and $\hat{\varphi}_c^{(2)}$ is the composite of universe score variance on the item side. Formally, these are defined as

$$\hat{\varphi}_d^{(2)} = \sum_d w_d^2 \hat{\varphi}_{2d}^{(2)} + \sum\sum_{d' \neq d} w_d^2 w_{d'}^2 \hat{\varphi}_{2dd'}^{(2)} \tag{30}$$

$$\hat{\varphi}_c^{(2)} = \sum_c w_c^2 \hat{\varphi}_{1c}^{(2)} + \sum\sum_{c' \neq c} w_c^2 w_{c'}^2 \hat{\varphi}_{1cc'}^{(2)} \tag{31}$$

where the weights $w_d = \frac{n_{id}}{n_I}$ for $d = 1,\ldots D$ and $w_c = \frac{n_{jc}}{n_J}$ for $c = 1,\ldots C–1$, n_I is the total number of items over all dimensions, n_{id} is the number of items in dimension d, n_j is the total number of items over all dimensions, and n_{jc} is the number of persons who got each category score.

Estimation

For generalized linear mixed models with crossed random effects, the likelihood of the data given the random variables needs to be integrated over the latent distribution. Since the high-dimensional likelihood function does not have a closed form in general, there are several approaches to approximating the maximum likelihood. The Laplacian approximation evaluates the unscaled conditional density at the conditional mode and is optimized with respect to the fixed effects and the disturbances. It is equivalent to the adaptive Gaussian quadrature with one node and is most accurate when the integrand of the likelihood is proportional to a normal density. Thus, a large cluster size corresponds to close-to-normal posterior density of the random variables, which then again leads to better approximation and less bias in estimates, especially for person parameter estima-

tion (Cho & Rabe-Hesketh, 2011; De Boeck et al., 2011; Joe, 2008; Pinheiro & Bates, 1995; Skrondal & Rabe-Hesketh, 2004).

Specifically, the model is fitted using the computational method implemented in the **lme4** package (Bates, 2010). Given the response vector $\mathcal{Y}$, the q-dimensional random effect vector $\mathcal{B}$, the variance-component parameter vector θ, the scale parameter σ for which it is assumed that $\sigma > 0$, and a multivariate Gaussian random variable $\mathcal{U}$ such that $\mathcal{B} = \Lambda_\theta \mathcal{U}$ where a covariance matrix Λ_θ satisfies

$$Var(\mathcal{B}) = \Sigma_\theta = \sigma^2 \Lambda_\theta \Lambda_\theta^\intercal, \tag{32}$$

the joint density function of $f_{\mathcal{U},\mathcal{Y}}(\boldsymbol{u}, \boldsymbol{y})$ is evaluated at the observed vector $\boldsymbol{y}_{obs}$. The continuous conditional density $f_{\mathcal{U}|\mathcal{Y}}(\boldsymbol{u}|\boldsymbol{y}_{obs})$ can be expressed as a function of an unnormalized conditional density $h(\boldsymbol{u})$, of which integral $\int f(\boldsymbol{u}) d\boldsymbol{u}$ is the same as the likelihood that needs to be evaluated for our model fitting.

Since the integral does not have a closed form for the kinds of mixed model we are interested in, it is evaluated using the Laplace approximation that utilizes the Cholesky factor $\boldsymbol{L}_\theta$ and the conditional mode $\tilde{\boldsymbol{u}}$. The conditional mode of u given $\mathcal{Y} = \boldsymbol{y}_{obs}$ is defined as a maximizer of the conditional density and a minimizer of a penalized residual sum of squares (PRSS) criterion or a function of the parameters given the data,

$$r_{\theta,\beta}^2 = \min_{\boldsymbol{u}} \|\boldsymbol{y}_{obs} - \mu\|^2 + \|\boldsymbol{u}\|^2, \tag{33}$$

where μ is the mean of the conditional density. The Cholesky factor $\boldsymbol{L}_\theta$ is defined as the sparse lower triangular $q \times q$ matrix with positive diagonal elements such that

$$\boldsymbol{L}_\theta \boldsymbol{L}_\theta^\intercal = \boldsymbol{\Lambda}_\theta^\intercal \boldsymbol{Z}^\intercal \boldsymbol{Z} \boldsymbol{\Lambda}_\theta + \boldsymbol{I}_q. \tag{34}$$

The sparse triangular matrix $\boldsymbol{L}_\theta$ can be efficiently evaluated even with large data sets by the fill-reducing permutation that reduces the number of non-zeros in the factor. After evaluating $\boldsymbol{L}_\theta$ and solving for $\tilde{\boldsymbol{u}}$, the likelihood can be conveniently expressed as a function of σ, $\boldsymbol{L}_{\theta,\beta}$, and $r_{\theta,\beta}^2$. On a deviance scale, the Laplace approximation of the likelihood is given as

$$d(\theta, \beta, \sigma | y_{obs}) = -2\log(L(\theta, \beta, \sigma | y_{obs})) \approx= n \cdot \log(2\pi\sigma^2) + 2 \cdot \log|\boldsymbol{L}_{\theta,\beta}| + \frac{r_{\theta,\beta}^2}{\sigma^2} \tag{35}$$

and the parameter estimates are the values at which this deviance is minimized.

Currently, xtmelogit in Stata and the lmer() function in R are as available as general statistical packages that have the capacity to estimate one or more cross-classified random variables using the Laplacian approximation, while the lmer() function is significantly more efficient than xtmelogit with regard to the calculation time. Therefore, we chose the lmer() function to estimate our model parameters.

In addition to the IRT random person and item variables, we also parameterize and estimate the variance components using lmer(). This direction also corresponds to the recommendation by Robinson (1991), Gelman, Carlin, Stern, & Rubin (2003), and Gelman (2005) to treat the hierarchical regression coefficients as random variables (and thus 'predicted') and the variance components as parameters (and thus 'estimated'). In traditional GT, since it depends on the analogous-ANOVA variance decomposition procedure based on the design of the existing data, there are known limitations such as negative variance estimates. ANOVA's main advantage is the ease of variance component estimation, but it is mostly applied to balanced designs. With proper reconstruction of data, lmer() easily estimates the variance components of incomplete data, which, we argue, would serve as a significant improvement of the problems in traditional GT. In addition, there has been no clearly best estimation method for variance decomposition of incomplete data and unbalanced mixed designs. Even though the resulting estimates have been proved to be unbiased, other properties of estimates are generally unknown (Khuri, 2000; Khuri & Sahai, 1985; Skrondal & Rabe-Hesketh, 2004). It is thus useful to know that for unbalanced multistrata ANOVA, lmer() is preferred to estimate variance components rather than the aov() and Anova() functions, which are also currently available in R for general ANOVA.

The key to estimating the generalizability coefficients lies in using proper variance component estimates for diverse measurement situations. We take the variance component estimates from lmer() and use the calculation methods from Brennan (2001), which includes the most comprehensive set of calculation methods for these coefficients in measurement situations that match a variety of complex ANOVA-like designs. Recall that the classical definition of reliability is the proportion of the total variance of the measurements that is due to the true score variance. We take this definition to calculate the generalizability coefficient $E(\hat{\rho}_j^2)$ for person measurement. In the same manner, we calculate the generalizability coefficient for measurement of item easiness and rater leniency as specified in Equations (20) to (22). In the multidimensional and/or polytomous case, the dimension-specific and category-specific variance components are estimated as specified in Equations (26) to (29).

Through extensive simulation studies (Choi, 2013), the accuracy of the results from the proposed approach in various measurement conditions was evaluated. In conclusion, the simulation results suggested that the proposed approach gives overall accurate generalizability coefficients. While more students and skewness in person distributions showed a significant interaction effect on the accuracy of the generalizability coefficients, the effect sizes were all very small. The next section presents the datasets and design of an empirical study.

The example data

The illustrative data set has three features that illuminate the utility of the proposed method: multidimensionality, polytomous responses, and multiple raters. The data was collected by researchers from Michigan State University and the University of Califor-

nia, Berkeley for the Carbon Cycle project, which was supported by the National Science Foundation: *Developing a Research-based Learning Progression on Carbon-Transforming Processes in Socio-Ecological Systems* (NSF 0815993). The participants included U.S. students from the state of Michigan in grades 4 through 12 during the 2008–2009 school year. After the data were cleaned, the data consisted of 869 students, including 190 elementary students, 346 middle school students, and 333 high school students. The 19 items in the data set represented six latent ability domains and were polytomously scored into four categories by 8 raters. The numbers of items designed to measure each of the dimensions were 3, 3, 3, 2, 3, 3, and 5, respectively. However, not every item was designed to measure all six domains, not every item was rated by all 8 raters, not every person answered all items, not every item had four category scores, and so on. That is, the data was unbalanced and incomplete. The reshaping of the data, based on Tutz's (1990) sequential stage continuation ratio logit, resulted in a response vector with a length of 18,695. A unidimensional model and a multidimensional model for polytomous data with a rater facet were fitted to this data set.

The models fitted to the Carbon Cycle 2008-2009 empirical data sets were (a) a unidimensional model (without raters), UD, (b) a unidimensional model (with raters), UDR, (c) a multidimensional model (without raters), MD, and (d) a multidimensional model (with raters), MDR. The composite person and item variance components are the weighted averages based on the number of items per each dimension and the number of persons who scored each category, respectively. The results are summarized in Table 1.

The person, item, and rater variance component estimates stay relatively stable across the four models. Overall, adding the rater effect to the unidimensional model (UD to UDR) and to the multidimensional model (MD to MDR) did not result in noticeable changes in the person and item variance component estimates and generalizability coefficients. The person generalizability coefficients decreased about 0.02 on average: from 0.340 and 0.249 to 0.315 and 0.224 for the unidimensional case, and from 0.376 and 0.286 to 0.352 and 0.261 for the multidimensional case. The item generalizability coefficients changed on average less than 0.005: from 0.358 and 0.269 to 0.360 and 0.274 for the unidimensional case, and from 0.337 and 0.241 to 0.335 and 0.243 for the multidimensional case.

Table 1:

Estimated Variance Components and Generalizability Coefficients

	Model			
	UD	UDR	MD	MDR
	Est	Est	Est	Est
Person	1.696	1.509	1.981(c)	1.783(c)
Dim 1			2.803	2.678
Dim 2			2.044	1.785
Dim 3			1.105	1.062
Dim 4			2.066	1.977
Dim 5			2.321	1.978
Dim 6			1.623	1.386
Item	1.836(c)	1.845(c)	1.669(c)	1.658(c)
Step 1	2.799	2.780	2.470	2.446
Step 2	0.639	0.659	0.541	0.559
Step 3	2.089	2.123	2.046	2.015
Rater		0.093		0.092
Error	3.287	3.287	3.287	3.287
AIC	12,168	12,130	12,177	12,140
BIC	12,246	12,216	12,451	12,422
Dev.	12,148	12,108	12,107	12,068
GCP	0.340	0.315	0.376	0.352
GCI	0.358	0.360	0.337	0.335
GCR		0.027		0.027
IDP	0.249	0.224	0.286	0.261
IDI	0.269	0.274	0.241	0.243
IDR		0.014		0.013

Notes. 1. The item and rater parameters are interpreted as easiness and leniency, respectively. 2. The (c) marks are for the weighted or composite variance component estimates.

Similarly, adding the multiple dimensions did not produce significantly different generalizability coefficients for person, item, and rater facets. Compared to the unidimensional results, the person-side generalizability coefficients were slightly greater (about a 0.035 increase on average) while the item-side generalizability coefficients were slightly smaller (about a 0.025 decrease on average). Including the multiple person dimensions in the model only slightly improved the deviances: from 12,148 to 12,107 for the models without the rater effect and from 12,108 to 12,068 for the models with the rater effect.

However, the AIC and the BIC showed that the multidimensional models did not fit better than the unidimensional models did. Thus, we can select the simpler unidimensional model when summarizing the generalizability coefficients for the Carbon Cycle data analysis. The best fit was found for the unidimensional model with the rater effect (UDR), where the generalizability coefficient and the index of dependability of the person measurements (0.315 and 0.224) were about 0.05 logit less than those of the item measurements (0.360 and 0.274). The rater variance component was very small (0.093) and the resulting generalizability coefficients for the raters were also very small (0.027 and 0.014).

Previous research on the multidimensionality of the same data using multidimensional item response models also reported high latent correlations between the dimensions, as shown in Table 2 (Choi, Lee, & Draney, 2009). All empirical results lead to the conclusion that the person dimensions are statistically indistinguishable.

We use the results from the polytomous multidimensional model with the rater effect to illustrate the advantages of using the proposed approach. While the model fit was worse than the unidimensional model, we purposefully chose this model because our goal here is to demonstrate the extendibility of the proposed approach to more complex test conditions. We can not only estimate random variance components of the persons, items, and raters but also estimate the individual person, item, and rater estimates via the proposed method. Table 3 shows examples of those predicted individual estimates. When the intercepts and group means are added, these estimates are comparable to the traditional item response theory person ability, item step difficulty (easiness with reversed sign), and rater severity (leniency with reversed sign) estimates on the same logit scale. For example, the fixed effects estimates for the grand mean and the dimension 2 were 1.998 and 0.177, respectively. Thus, the estimated ability for the dimension 2 of the student S00001 is 1.998 + 0.177 – 2.623 = –0.448 logit.

Table 2:

The correlation between six person dimensions for the 2008-2009 Carbon Cycle data

	Dimension 1	Dimension 2	Dimension 3	Dimension 4	Dimension 5
Dimension 2	0.994				
Dimension 3	0.989	0.999			
Dimension 4	0.857	0.901	0.911		
Dimension 5	0.962	0.984	0.988	0.904	
Dimension 6	0.992	1.000	1.000	0.911	0.983

Next, Figure 1 and Figure 2 present the precision in predicting the random person ability, item difficulty, and rater severity effects. First, in Figure 1, the students and items are ordered from left to right according to increasing standard normal quantiles. The dots are the conditional modes of the random effects, and the lines indicate the 95% prediction intervals. The prediction interval is obtained from the conditional standard deviation,

which is a measure of the dispersion of the parameter estimates given the data (Bates et al., 2015). The x axis is the standard normal quantiles for students and items and the y axis is the logit scale. The patterns show that about 40% to 50% of the person random effects contain zero in their prediction interval while most of the item random effects do not. This means that for the students it is more probable that their ability estimate is close to the mean than for the items. This is reasonable because the cluster sizes for person estimation are much smaller than those for items.

Figure 2 depicts the pattern of the 95% prediction intervals on the person and item random effects ordered differently according to increasing estimated values for the first level (e.g., dimension 1 for persons, step 1 for items). By doing so, we can discern whether the patterns of the dimensions are similar to each other. The x axis is the logit scale and the y axis is the persons or the items, respectively. The graph confirms that the person dimensions are highly correlated with the dimension 1 except for dimension 4, as shown by the previous results of the latent correlation from a multidimensional IRT analysis shown in Table 2.

Table 3:

An example set of predicted person, item, and rater effects estimates

Student ID	Dim 1	Dim 2	Dim 3	Dim 4	Dim 5	Dim 6
S00001	-3.178	-2.623	-2.029	-2.257	-2.679	-2.318
S00002	-0.211	-0.161	-0.102	-0.017	-0.138	-0.136
S00003	-1.141	-0.925	-0.688	-0.647	-0.900	-0.808
S00004	-0.202	-0.191	-0.187	-0.377	-0.159	-0.170
S00005	-1.578	-1.307	-1.019	-1.166	-1.237	-1.144
S00006	-1.441	-1.144	-0.811	-0.583	-1.182	-1.002
S00007	-1.885	-1.564	-1.223	-1.418	-1.643	-1.389
S00008	-1.405	-1.179	-0.944	-1.186	-1.151	-1.040
S00009	-2.677	-2.186	-1.651	-1.665	-2.222	-1.924
S00010	-2.593	-2.128	-1.625	-1.718	-2.197	-1.880
S00011	-0.977	-0.821	-0.658	-0.831	-0.776	-0.721
S00012	-1.597	-1.289	-0.951	-0.855	-1.193	-1.118
S00013	-1.441	-1.210	-0.968	-1.215	-1.353	-1.087
S00014	-1.277	-1.076	-0.867	-1.113	-1.048	-0.949
S00015	-0.853	-0.663	-0.446	-0.207	-0.571	-0.563
⋮	⋮	⋮	⋮	⋮	⋮	⋮

Note. All students shown here are at elementary school level, hence the low estimates.

Item ID	Step 1	Step 2	Step 3
i01	0.949	0.092	0.960
i02	-2.519	-1.355	-0.278
i03	-0.842	0.268	0.302
i04	-0.715	-0.903	0.061
i05	1.307	0.308	0.755
i06	1.128	0.901	0.215
i07	-2.832	-0.916	-3.245
i08	-0.772	-0.166	0.521
i09	1.255	0.558	0.672
i10	-1.354	-0.802	0.157
i11	-0.148	-0.001	0.726
i12	0.212	-0.588	1.121
i13	-0.198	-0.092	2.055
i14	0.020	0.321	-0.271
i15	1.759	0.611	1.592
i16	-0.155	-0.166	2.304
i17	-1.167	0.250	-2.576
i18	-1.998	-1.529	-2.813
i19	1.803	0.971	0.557

Rater ID	Estimates
r1	-0.014
r2	0.200
r3	0.014
r4	0.270
r5	-0.005
r6	-0.240
r7	-0.551
r8	0.182

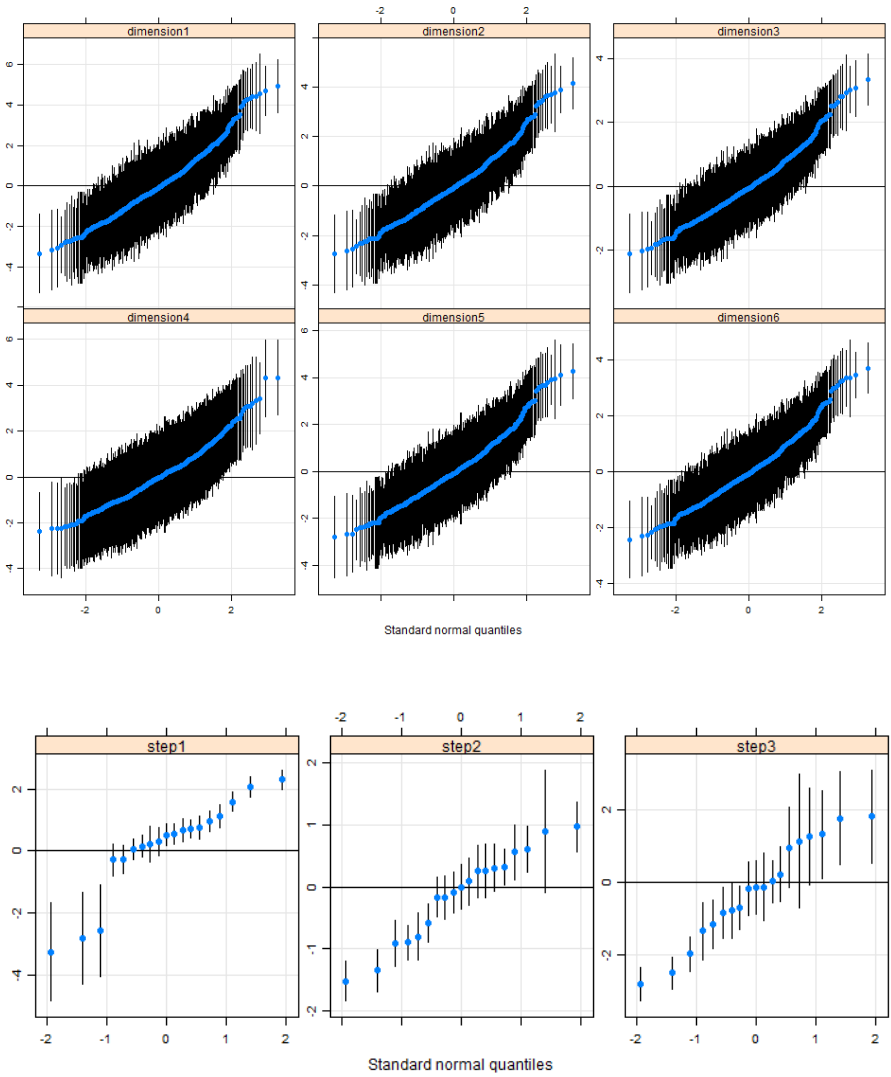


Figure 1:

95% prediction intervals on the person and item random effects compared to the standard normal quantiles of students and items

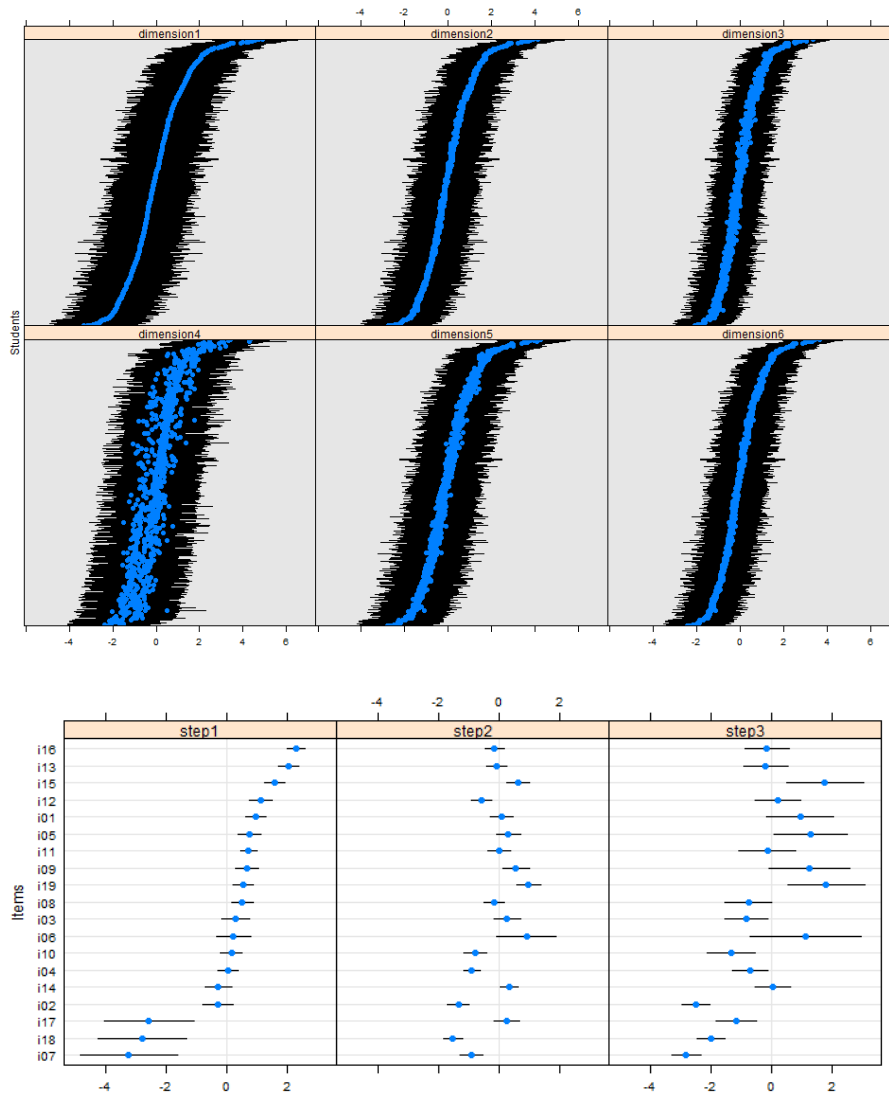


Figure 2:

95% prediction intervals on the person and item effects ordered based on increasing estimated values for the first levels (e.g., dimension 1 and step 1)

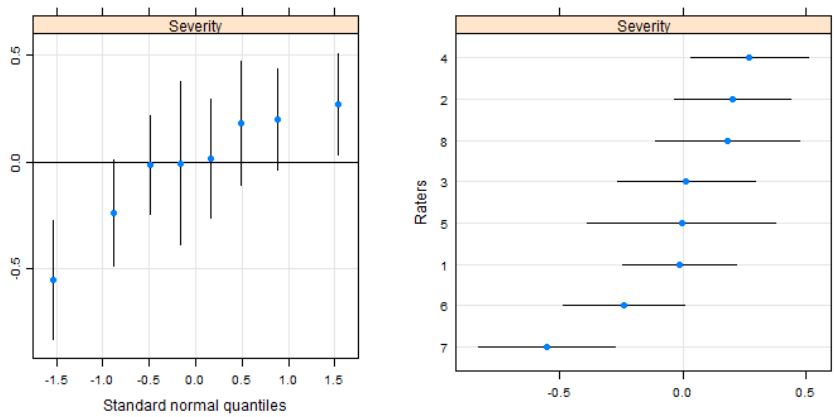


Figure 3:

95% prediction intervals on the rater effects ordered based on 1) the standard normal quantiles and 2) increasing estimated values

On the other hand, the item step estimates are not showing much correlation among the steps. What is more interesting in the item graphs is that we can observe for some items (e.g., the first three items from the bottom: i7, i17 and i18), achieving the second and the third steps was relatively easier than for other items. The greater imprecision (i.e., longer bars crossing the estimate) for the first step is caused by the small number of responses at that level of performance, compared to the responses at higher levels of performances which showed greater precision (i.e., shorter bars).

The 95% prediction intervals for the rater random effects are shown in Figure 3. In the graph on the left, the x axis is the standard normal quantiles and the y axis is the logit scale. Overall, the rater estimates are quite close to each other and to zero, as we should expect, since the raters went through training and screening procedures. Unlike the person and item estimates, the y scale ranges narrowly between -0.5 and 0.5 logits. Only two rater estimates do not have prediction intervals that contain zero. In the graph on the right, the x axis is the logit scale and the y axis is the persons or the items. The rater 7 was the most lenient: when other variables were controlled, it was easier for the students to get the scores on items when rated by the rater 7.

The benefit of having the set of predicted person, item, and rater effects in Table 3 is explicitly shown through Figure 4, a modified version of a Wright Map (Wilson, 2005). The person ability estimates for each dimension are calculated as sums of the estimated intercept, the cluster mean for each dimension (except the reference dimension 1), and the estimated person random effects. Likewise, the item difficulty estimates for each step are calculated as sums of the estimated intercept, the cluster mean for each step (except for the reference step 1), and the estimated item random effects. The rater severity estimates were calculated as the means of the rater severity for the three thresholds. One can compare the location of the distributions of the person, item, and rater parameter esti-

mates on the same logit scale, which gives insight into how easy it was for the students to get the target score on the items by the raters.

The person ability scores are displayed as density curves on the logit scale. Dimensions 3 and 4 have relatively small variances and the shape of dimension 4 density is slightly off from the symmetric normal distribution. The ability distributions of the six dimensions shared similar locations and dispersions, again validating the finding that the multidimensional model does not fit the data better than the unidimensional one. The result corresponds with the lower latent correlation for dimension 4 with other dimensions as well as the fuzzy dots in the 95% prediction interval graph. Most likely, the reason why dimension 4 behaves as an oddity is the very low number of items (2) that measures it.

Next, the three groups of points represent item difficulties. The location of the points can be interpreted as where the students on average have a 50% probability to get a particular score as compared to a score below that. Since the data had four possible scores (0,1,2,3), three steps or thresholds exist between the four scores. As the Wright Map shows, most of the students had more than 50% chance to achieve the first thresholds of all items, except for the items 13 to 16. In other words, for most of the students, getting the score 1 versus 0 was relatively easy. The second thresholds of the items were reasonably located near the means of the person distributions, meaning that on average students were able to get the score 2 on most items with about a 50% probability of success. Getting the highest score was generally difficult for the students, particularly for the items 1, 5, 6, 9, and 15.

Last, the raters were generally non-separable from each other except for the rater 6 and 7 who were on average more lenient than others in giving scores (compared to the score below) for the items to the students. The small variance component, the small resulting generalizability coefficient, and non-separable individual rater effects that we found in this analysis suggest that the rater training sessions were highly effective. The eight raters were indeed graduate research assistants who were involved in every stage of the research process — thus for this group of raters it makes sense that they showed consistent ratings.

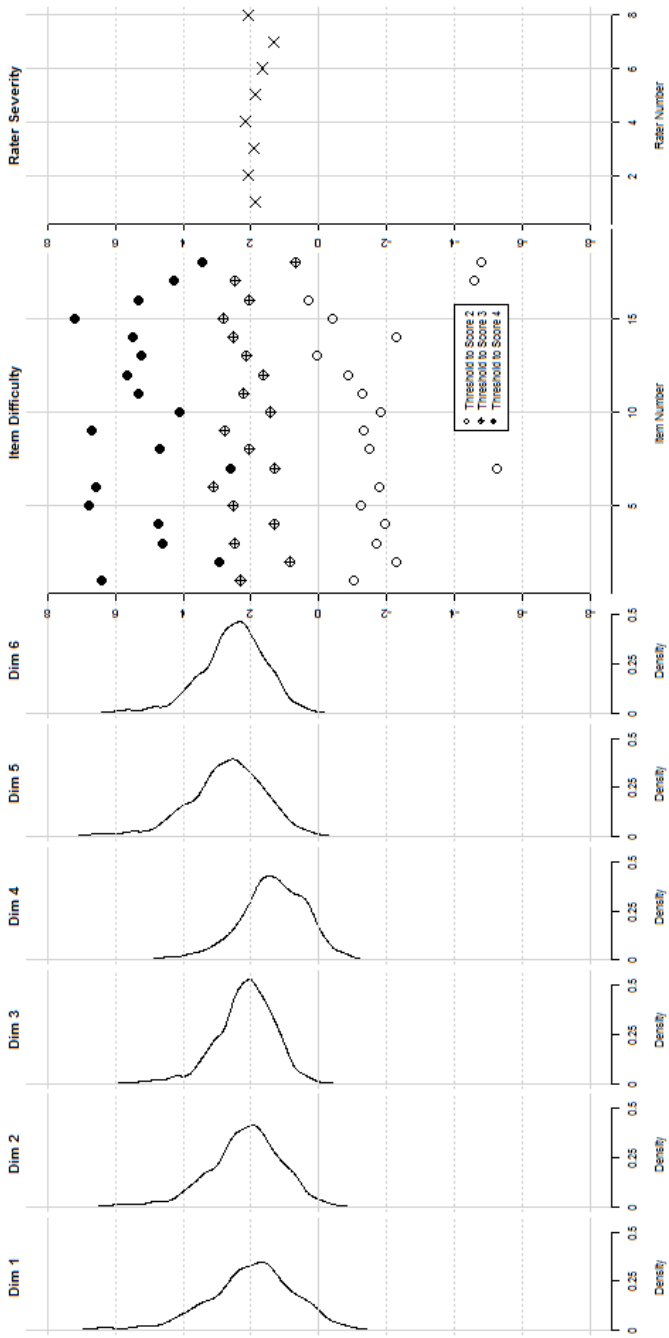


Figure 4:
Wright Map of person ability, item difficulty, and rater severity estimates

Discussion

In this study, we have suggested an approach for combining GT and IRT. We recognize that IRT models can be written in terms of a latent continuous response and that a classic IRT model can be modeled directly using a simple GT design with items as a fixed facet. The resulting logistic mixed models extend a classic IRT model by treating items as a random facet and/or considering other facets such as raters. The advantage of the proposed approach is that it allows a straightforward maximum likelihood estimation of individual random effects as well as the variance components needed for the generalizability coefficients.

In addition, application of the proposed approach was illustrated using a moderately large-scale education data set. The results demonstrated another advantage of the proposed approach: its flexibility with respect to incorporating extra complications in measurement situations (e.g., multidimensionality, polytomous responses) and explanatory variables (e.g., rater facet). The variance components and the generalizability coefficients were presented. Also, predicted individual random effects were presented by taking advantage of the usefulness of a modified Wright Map.

The results motivate further research on the following. In suggesting an approach to combine GT and IRT, the robustness of the generalizability coefficient estimates may not necessarily become a concern. However, the effects of (a) the discrete nature of data (e.g., more than two categories), (b) the violation of normality assumptions, and (c) more complex designs (e.g., person by item by rater design, multidimensionality), on the estimation accuracy of the variance components and the generalizability coefficients, should be examined and reported.

The proposed approach can be generalized to other measurement situations, both simpler and more complex ones. A simpler example is a balanced design with no missing data, or a design where the facets are nested. A more complex example is an unbalanced design with more than three crossed facets. For example, in addition to person, item, and rater facets, one could include an occasion facet that involves repeated measurement. Such attempts may offer an even closer connection between existing GT designs and IRT models. Currently, research is underway to extend the proposed approach to such alternative designs. It is in our hopes that the results from these studies will provide a more comprehensive basis to understand and evaluate methodological advantages and disadvantages of the existing and proposed approaches.

In the meantime, whether the proposed method is extendable to different designs, such as nested designs or designs with more than three facets, partly depends on the estimation methods chosen. Until recently, estimation of crossed random effects models has been limited to a relatively small number of random effects, or facets, and their levels. Even though the flexibility of the proposed approach allows a straightforward extension of the models to those situations, questions remain regarding how to estimate the variance components in the models with an increased number of crossed random facets. Moreover, incorporating other item parameters such as discrimination differences or guessing in IRT models may add more challenges in estimation and interpretation. It will be inter-

esting to investigate what advanced and/or alternative estimation methods might be needed in extending the approaches to combine GT and IRT.

Lastly, an interesting topic for further studies exists around understanding the interaction effect between raters and persons (i.e., ratees). For example, a rater's rating of a person's response can differ systematically based on the characteristics of the response that the person gave. An interaction can also exist between the persons' group membership and the raters. For example, some raters might rate female students' responses differently than male students' responses. Or raters might differ their ratings systematically between groups of students, not knowing which group each student belongs to. Jin & Wang (2017) recently discussed a mixture facets model to account for differential rater functioning — the interaction between the ratees' unknown group membership and raters. In the proposed approach the interaction between individual raters and individual persons can be either included or not included, although it was not the focus of this study to fully explore this topic. As interactions between these facets can occur in real testing situations, it will be worthwhile to further explore how best we can model this effect.

Conclusion

The integrated modeling approach provides advantages by combining GT and IRT analyses. The logistic mixed model allows for a straightforward and effective maximum likelihood estimation of individual random effects for IRT analysis as well as the variance components needed for GT analysis. Through the Laplacian approximation implemented in the lmer() function in R, it estimates more than one cross-classified random effect efficiently with regard to the calculation time; and it estimates the variance components of incomplete data from the unbalanced mixed design relatively easily, without producing negative variance estimates. The findings from the sample data analysis showed that the proposed approach can be extended to more complicated test conditions (e.g., multidimensionality, polytomous responses, multiple raters) and produces individual estimates for persons, items, and rater random effects as well as the generalizability coefficients for person, item, and rater facets.

References

Agresti, A., & Kateri, M. (2011). Categorical data analysis. In *The International Encyclopedia of Statistical Science* (pp. 206-208). Berlin: Springer.

Andrich, D. (1978). A rating formulation for ordered response categories. *Psychometrika, 43*(4), 561-573.

Bates, D. M. (2010). *lme4: Mixed-effects modeling with R.* URL http://lme4.r-forge.r-project.org/book.

Bates, D., Mächler, M., Bolker, B., & Walker, S. (2014). *Fitting linear mixed-effects models using lme4*. arXiv preprint arXiv:1406.5823.

Brennan, R. L. (2001). *Generalizability theory*. New York, NY: Springer.

Breslow, N. E., & Clayton, D. G. (1993). Approximate inference in generalized linear mixed models. *Journal of the American Statistical Association, 88,* 9–25.

Briggs, D. C., & Wilson, M. R. (2007). Generalizability in item response modeling. *Journal of Educational Measurement, 44*(2), 131–155.

Chien, Y. M. (2008). *An investigation of testlet-based item response models with a random facets design in generalizability theory.* Doctoral dissertation. The University of Iowa.

Cho, S. J., & Rabe-Hesketh, S. (2011). Alternating imputation posterior estimation of models with crossed random effects. *Computational Statistics & Data Analysis, 55*(1), 12-25.

Choi, J. (2013). *Advances in combining Generalizability Theory and Item Response Theory*. Doctoral dissertation. University of California, Berkeley: Berkeley, CA.

Choi, J., Briggs, D. C., & Wilson, M. R. (2009, April). *Multidimensional extension of the generalizability in item response modeling (GIRM)*. Paper presented at the 2009 National Council on Measurement in Education Annual Meeting, San Diego, CA.

Choi, J., Lee, Y.-S. & Draney, K. (2009). *Principle-based and process-based multidimensionality and rater effects in validation of the carbon cycle learning progression*. Paper presented at the American Educational Research Association Annual Meeting, San Diego, CA.

Cronbach, L. J., Gleser, G. C., Nanda, H., & Rajaratnam, N. (1972). *The dependability of behavioral measurements: Theory of generalizability for scores and profiles.* New York, NY: Wiley.

De Boeck, P., Bakker, M., Zwitser, R., Nivard, M., Hofman, A., Tuerlinckx, F., & Partchev, I. (2011). The estimation of item response models with the lmer function from the lme4 package in R. *Journal of Statistical Software*, *39*, 1–28.

Fahrmeir, L., & Tutz, G. (2001). *Multivariate statistical modeling based on generalized linear models* (2nd ed.). New York, NY: Springer.

Gałecki, A., & Burzykowski, T. (2013). *Linear mixed-effects models using R: A step-by-step approach*. New York, NY: Springer.

Gelman, A. (2005). Analysis of variance — why it is more important than ever. *Annals of Statistics, 33*(1), 1-53.

Gelman, A., Carlin, J. B., Stern, H. S., & Rubin, D. B. (2003). *Bayesian data analysis*. Boca Raton, FL: Chapman & Hall/CRC.

Iramaneerat, C., Yudkowsky, R., Myford, C., & Downing, S. (2008). Quality control of an OSCE using generalizability theory and many-faceted Rasch measurement. *Advances in Health Sciences Education, 13*(4), 479–493.

Jin, K. Y., & Wang, W. C. (2017). Assessment of differential rater functioning in latent classes with new mixture facets models. *Multivariate Behavioral Research, 52*(3), 391–402.

Joe, H. (2008). Accuracy of Laplace approximation for discrete response mixed models. *Computational Statistics and Data Analysis, 52*, 5066–5074.

Khuri, A. I. (2000). Designs for variance components estimation: Past and present. *International Statistical Review, 68*(3), 311-322.

Khuri, A. I., & Sahai, H. (1985). Variance components analysis: A selective literature survey. *International Statistical Review/Revue Internationale de Statistique*, 279-300.

Kolen, M., & Harris, D. (1987, April). *A multivariate test theory model based on item response theory and generalizability theory*. Paper presented at the annual meeting of the American Educational Research Association, Washington, DC.

Linacre, J. M. (1993, April). *Generalizability theory and many-facet Rasch measurement*. Paper presented at the annual meeting of the American Educational Research Association, Atlanta, GA.

Lord, F. M. (1980). *Applications of item response theory to practical testing problems*. Hillsdale, NJ: Erlbaum.

Lynch, B. K., & McNamara, T. F. (1998). Using G-theory and many-facet Rasch measurement in the development of performance assessments of the ESL speaking skills of immigrants. *Language Testing, 15*(2), 158-180.

MacMillan, P. D. (2000). Classical, generalizability, and multifaceted Rasch detection of interrater variability in large, sparse data sets. *Journal of Experimental Education, 68*, 167–190.

Masters, G. N. (1982). A Rasch model for partial credit scoring. *Psychometrika, 47*(2), 149-174.

Patz, R. J., Junker, B. W., Johnson, M. S., & Mariano, L. T. (2002). The hierarchical rater model for rated test items and its application to large-scale educational assessment data. *Journal of Educational and Behavioral Statistics*, *27*(4), 341–384.

Pearson, K. (1901). Principal components analysis. *The London, Edinburgh and Dublin Philosophical Magazine and Journal, 6*(2), 566.

Pinheiro, J. C., & Bates, D. M. (1995). Approximations to the log-likelihood function in the nonlinear mixed-effects model. *Journal of Computational and Graphical Statistics, 4*(1), 12-35.

R Core Team (2017). *R: A language and environment for statistical computing*. R Foundation for Statistical Computing, Vienna, Austria. URL https://www.R-project.org/.

R Development Core Team (2011). *R: A language and environment for statistical computing*. R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0, URL http://www.r-project.org/.

Rabe-Hesketh, S., Skrondal, A., & Pickles, A. (2004). Generalized multilevel structural equation modeling. *Psychometrika, 69*(2), 167-190.

Rasch, G. (1960). *Probabilistic models for some intelligence and attainment tests*. Copenhagen, Denmark: Denmarks Pedagogiske Institut.

Robinson, G. K. (1991). That BLUP is a good thing: The estimation of random effects. *Statistical Science*, 15-32.

Samejima, F. (1997). Graded response model. In W. J. van der Linden & R. K. Hambleton (Eds.), *Handbook of modern item response theory* (pp. 85-100). New York: Springer.

Skrondal, A., & Rabe-Hesketh, S. (2004). *Generalized latent variable modeling: Multilevel, longitudinal, and structural equation models.* Boca Raton, FL: Chapman & Hall/CRC.

Smith, E. V., & Kulikowich, J. M. (2004). An application of generalizability theory and many-facet Rasch measurement using a complex problem solving skills assessment. *Educational and Psychological Measurement, 64,* 617–639.

Sudweeks, R. R., Reeve, S., & Bradshaw, W. S. (2005). A comparison of generalizability theory and many-facets Rasch measurement in an analysis of college sophomore writing. *Assessing Writing, 9*(3), 239–261.

Tutz, G. (1990). Sequential item response models with an ordered response. *British Journal of Mathematical and Statistical Psychology, 43,* 39–55.

Wilson, M. (2005). *Constructing measures: An item response modeling approach*. Mahwah, NJ: Erlbaum.

Wright, B. D., & Stone, M. A. (1979). *Best test design.* Chicago, IL: MESA Press,

Wu, M. L., Adams, R. J., Wilson, M. R., & Haldane, S. A. (2007). ACER ConQuest version 2.0: Generalised item response modelling software [Computer software and manual]. Camberwell, Australia: ACER Press.

Zheng, X., & Rabe-Hesketh, S. (2007). Estimating parameters of dichotomous and ordinal item response models with gllamm. *Stata Journal, 7*(3), 313-333.

Appendix

We provide generic examples of lmer() syntax for logistic mixed models with crossed random effects. In the syntax, we assume that the name of dataset is 'data'. Syntax for simpler models have been also provided for comparisons. We recommend Gałecki & Burzykowski (2013) for details in specifying linear mixed effects models using R. Please contact authors for further assistance with model specification.

(a) Main effects for persons, items, and raters

```
R> lmer(y ~ personid + itemid + raterid, data=data,
   family=binomial)
```

(b) Main effects with random intercepts for persons, items, and raters

```
R> lmer(y ~ (1|personid) + (1|itemid) + (1|raterid),
   data=data, family=binomial)
```

(c) Crossed random effects with random intercepts for persons, items, raters and their interactions

```
R> lmer(y ~ (1|personid) + (1|itemid) + (1|raterid) +
   personid:itemid + personid:raterid + per
   sonid:itemid:raterid , data=data, family=binomial)
```

Psychological Test and Assessment Modeling, Volume 60, 2018 (1), 81-100

Comparison of human rater and automated scoring of test takers' speaking ability and classification using Item Response Theory

Zhen Wang[1] *& Yu Sun*[2]

Abstract

Automated scoring has been developed and has the potential to provide solutions to some of the obvious shortcomings in human scoring. In this study, we investigated whether SpeechRater[SM] and a series of combined SpeechRater[SM] and human scores were comparable to human scores for an English language assessment speaking test. We found that there were some systematic patterns in the five tested scenarios based on item response theory.

Keywords: SpeechRater[SM], human scoring, item response theory, ability estimation and classification

[1]*Correspondence concerning this article should be addressed to:* Zhen (Jane) Wang, Educational Testing Service, Senior Psychometrician, Psychometrics, Statistics and Data Sciences (PSDS), Rosedale-Anrig, Princeton, NJ, U.S.A; email: jwang@ets.org.

[2]Senior Psychometric Analyst, Psychometrics, Statistics and Data Sciences (PSDS), Rosedale-Anrig, Princeton, NJ,

Language testing organizations in the United States routinely deal with large test taker samples, especially for certain Asian, European, and mid–eastern countries. For example, it is not unusual to have more than a million test takers worldwide taking a certain test in a given year, with each test taker responding to six items producing a total of more than six million responses. While having large samples is certainly not exclusive to language testing, constructed response item scoring, including essay scoring and spoken response scoring, is definitely an added complication for scoring.

Human scoring must be closely monitored within each administration and across administrations to ensure the quality and consistency of the human ratings. The effects of differences between human raters may substantially increase the bias in test takers' final scores without careful monitoring (Wang & Yao, 2013). This makes human scoring very labor intensive, time consuming and expensive (Zhang, 2013). The importance of these language tests for relatively high stake decisions places a lot of pressure on the entire system to ensure accurate scoring and consistent ratings on demand.

Automated scoring has been developed and has the potential to provide solutions to some of the obvious shortcomings in human scoring (e.g., rater inconsistency; rater drift; inefficiency). Bennett and Bejar (1998) indicated that automated scoring procedures allow for the scoring rules to be applied consistently. Automated scoring has some advantages including "*fast scoring, constant availability of scoring, lower per unit costs, greater score consistency, reduced coordination efforts for human raters, and potential for a degree of performance specific feedback*" (Ramineni, Trapani, Williamson, Davey, & Bridgeman, 2012). Recently, more and more operational scoring programs have started using automated scoring to augment human scorers. In the literature, we found some studies on the comparison of human and automated scoring conducted in recent years (Attali, Bridgeman & Trapani, 2010; Attali & Burstein, 2006; Chodorow & Burstein, 2004; Laundauer, Laham, & Foltz, 2003; Nichols, 2005; Ramineni et al., 2012; Streeter, Bernstein, Foltz, & Deland, 2011; Wang & von Davier, 2014; Wang, Zechner & Sun, 2016; Williamson, Xi, & Breyer, 2012). Wang et al. (2016) conducted a comprehensive analysis of SpeechRater^{SM} and human scoring at item, rater and form levels based on the speaking data from 10 administrations. For automated scoring, there is no need to examine changes in rater severity. Automated scoring is not prone to these types of changes. However, if an engine update occurs, the scores need to be investigated for comparability against human rater scores. As Attali (2013) pointed out, the relation of automated scores to human scores provides important information about the validity of automated scores.

In terms of operational scoring, several researchers have called for the need to study how to combine human and machine scores (Attali, 2013; Zhang, Breyer, & Lorenz, 2013), which is lacking in the current research literature. Due to the shortcomings of human rater and automated scoring, the argument of combining the two is appealing. The shift to the combination of the two allows us to move beyond using human scoring as the default "gold standard" and become less concerned about the construct underrepresentation of the automated scoring.

The primary purpose of the present research is to investigate whether human raters and SpeechRater^{SM} scores and different combinations of the two are comparable for test takers' final scoring, ability estimation, and classification.

Research questions

The major research question in this study is whether SpeechRater[SM] and a series of combined SpeechRater[SM] and human scores are comparable to human scores for an English language assessment speaking test. The current study targeted the following specific research questions:

Do the scores from SpeechRater[SM] only, Human Raters only, or different combinations of the two result in parallel[1], tau–equivalent[2], or congeneric[3] test scores for the final scoring of the speaking test?

Do the scores from SpeechRater[SM] only, Human Raters only, or the different combinations of the two result in similar IRT parameter estimates (item difficulty and discrimination) and test takers' ability estimates of the population values and classifications?

Method

Data

The speaking section of the English language assessment used in this study elicits a total of 5.5 minutes of speech for a candidate: two independent items that ask test takers to talk for 45 seconds on a familiar topic (e.g., "Describe a person that you admire."), and four integrated items in which reading and/or listening stimuli are presented first, and then the test taker has one minute to respond to each prompt based on these stimuli.

Each response to a speaking item is scored holistically by a single trained human rater on a 4-point discrete scale of 1–4, with "4" indicating the highest proficiency level and "1" the lowest. The scores are assigned based on rubrics, one each for independent and integrated items. The rubrics describe the aspects of the speaking construct that are deemed most relevant for determining the speaking proficiency of test takers and thus guide human raters in their scoring decisions. Each score level has a description of prototypical observed speaking behavior in three main areas of spoken language: delivery (fluency and pronunciation), language use (vocabulary and grammar aspects), and topic development (progression of ideas and content relevance). Human raters usually get "batches" of responses for a particular prompt (rather than scoring, e.g., all the responses of one candidate). In addition, a random sample of about 10% of responses in each administration is scored by a second human rater for reliability control purposes. If the two scores disagree by more than one point, a third rater is asked to adjudicate the score. Finally, the six-item scores are aggregated and scaled for score reporting purposes.

Data were drawn from 10 administrations involving 110 countries in 2012–2013. Among the 10 administrations, half of them were mainly from the western hemisphere and the other half were mainly from the eastern hemisphere. We randomly sampled 1100 test takers per administration. The speaking section of the English language assessment consists of six items. This yields a total of $10 \times 1100 \times 6 = 66{,}000$ responses that were scored by the SpeechRater[SM] engine. We pulled the first human rater scores (H1-rater), including

second human rater scores (H2-rater, if available), from a data repository (note that "H1" and "H2" are logical labels for human raters; in actuality, each of "H1" scores and "H2" scores comprise scores from a large number of human raters.) As stated above, H2-rater scores were only available for 10% of the data, which is a random sample from the administrations selected for reliability purposes.

During the operational cycle, all human raters (both H1-rater and H2-rater) participated in a standardized training process before they were allowed to rate the speaking items. In this study, we focused on the comparison of the item scores between the H1-rater and SpeechRater^{SM}. The H2-rater was from the same rater pool as the H1-rater, so there should not be any systematic differences between the results from the H1-rater and the H2-rater. We also made comparisons between the scores assigned by the H1-rater and the H2-rater for the 10% reliability sample.

In addition to the main data set used for this study (66,000 spoken responses), we used 10,000 spoken responses to items in other forms of the same assessment to estimate the parameters of the linear regression model used by SpeechRater^{SM} (discussed in next section). A separate data set of 52,200 responses from the same assessment was used for training the parameters of the ASR system.

SpeechRater^{SM}

SpeechRater^{SM} is an automated scoring engine developed at ETS that has been used in the TOEFL Practice Online program since 2006. It consists of an automatic speech recognizer (ASR system), feature computation modules, filtering model, a multiple regression scoring model to predict scores produced by human scorers for each spoken response (Zechner, Higgins, Xi, & Williamson, 2009). The filtering model, used to filter out non-scorable responses, is an important component of SpeechRater^{SM} (Higgins, Xi, Zechner, & Williamson, 2011). In recent years, the coverage of the speaking construct has been substantially extended from its original focus on fluency, pronunciation, and prosody by adding features related to vocabulary, grammar, and content, among others (Chen & Zechner, 2011; Xie, Evanini, & Zechner, 2012; Yoon & Bhat, 2012; Yoon, Bhat, & Zechner, 2012).

Data analyses

Classical analyses

Score comparisons using classical analysis were made based on different scoring scenarios. Analyses were conducted to answer the question of whether scoring based on the SpeechRater^{SM} only, human raters only, or combinations of the two would result in parallel, tau-equivalent, or congeneric test scores for the final scoring of the speaking test.

Different scoring scenarios tested include the following five scores:

S1. Human rater (H1-rater) only;

S2. SpeechRater^{SM} only;

S3. Total of SpeechRater^{SM} and human rater;

S4. Lower weighting (1/3) on human and higher weighting (2/3) on SpeechRater^{SM};

S5. Higher weighting (2/3) on human and lower weighting (1/3) on SpeechRater^{SM}.

The above scenarios can also be expressed as different weights in the weighted sum of the two ratings (human rater and SpeechRater^{SM}, respectively):

S1. (1, 0);

S2. (0, 1);

S3. (1/2, 1/2);

S4. (1/3, 2/3);

S5. (2/3, 1/3).

The one-factor measurement model was initially tested within the linear structural equation framework of Jöreskog and Sörbom (1984). A single-factor measurement model was fit to the data, in which all the five speaking scores were set to load on the same common latent variable representing the English speaking ability construct.

The five different scoring scenarios (S1 – S5 above) for the final speaking scores, which are the aggregated score across all six speaking items, were considered as five measures (S1 – S5), each measure was based on the same speaking items in the test. Reliability was calculated for each of the scoring scenarios. Correlations were also calculated among the five measures based on the different scoring scenarios. Parallel measures are the most restrictive model, assuming equal true score variances and equal error variances. Tau-equivalent measures have equal true score variances, but possibly different error variances whereas congeneric models allow true score variances as well as error variances to differ. Covariance matrices were analyzed and weighted least squares estimates of the parameters were obtained. If the fit of the model to the data becomes worse as the model is made more restrictive, then the constraints are not plausible for the data (Maruyama, 1998).

Three fit indices were used to evaluate the model-data fit, root mean squared error approximation (RMSEA), comparative fit index (CFI), and the normed fit index (NFI). Some empirical guidelines were followed when evaluating these fit indices: a model with an RMSEA value below .08, a CFI value above .90, a NFI value above .90, were considered to be an acceptable fit; a model with an RMSEA value below .05 and a CFI (and NFI) value above .95 were considered a good fit (Browne & Cudeck, 1993; Hooper, Coughlan, & Mullen, 2008; & Hu & Bentler, 1999).

IRT analyses

Generalized partial credit model.

PARSCALE (Muraki & Bock, 1997) was used to analyze the speaking tests based on the Generalized Partial Credit Model (GPCM, Muraki, 1992). The normal distribution was used as the prior distribution of ability. The method employed in the calibration phase of PARSCALE is that of random-effects estimation through marginal maximum likelihood. The random-effects solution employs the EM method (estimation and maximization) of solving the marginal likelihood equations. We set the number of EM cycles to be 50, and the number of quadrature points as 41 in the EM and Newton estimation. The scale parameter was set at 1.7 in the PARSCALE run.

Each speaking item was analyzed with IRT models with the following scoring designs: a) with human rater only; b) with SpeechRater^{SM} only; c) with the sum of human and SpeechRater^{SM}; d) with different weighting: 1/3 of human and 2/3 of SpeechRater^{SM}; and e) with different weighting: 2/3 of human and 1/3 of SpeechRater^{SM}. A key goal of this study was to compare test takers' ability estimates, and IRT parameters from different scoring designs. To compare test takers' ability estimates and IRT parameters, we used squared bias and the mean squared error.

Root mean squared error (RMSE), absolute mean bias (ABSBIAS), and bias were computed for all parameters and were used to examine the IRT parameter estimate differences, such as ability, difficulty, and discrimination. For example, let f_{ref} be the ability parameter based on the scoring with human rater 1 only (reference) and let f_j be the SpeechRater^{SM} only or the combination of the two (see S1 to S5 above), then

$$RMSE = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(f_j - f_{ref})^2} \tag{1}$$

where n is the number of test takers. Here f represents ability parameter estimates.

The bias and absolute mean bias are defined as the following:

$$BIAS = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(f_j - f_{ref})} \tag{2}$$

$$ABSBIAS = \frac{1}{n}\sum_{j=1}^{n}\left|(f_j - f_{ref})\right| \tag{3}$$

Bias and absolute mean bias were used to assess the parameter estimate differences. The squared differences and squared bias were also calculated and plotted to show results of parameter estimate differences and comparison among different scoring scenarios.

Results

Research question 1

Do the scores obtained from SpeechRater^{SM} only, Human Raters only, or the different combinations of the two result in parallel, tau-equivalent, or congeneric test scores for the final scoring of the speaking test?

Correlations.

Correlations were calculated (see Table 1) among the five scores (S1 – S5) based on different scoring scenarios (H1-rater only; SpeechRater^{SM} only; sum of SpeechRater^{SM} and H1-rater; lower weighting on human and higher weighting on SpeechRater^{SM}; higher weighting on human and lower weighting on SpeechRater^{SM}). The lowest correlation (.759) was found between the scores from H1-rater only and SpeechRater^{SM} only, and the other correlations range from .866 to .994.

Table 1:
Correlations among Five Scoring Scenarios (S1 – S5)

	H1 only	SR only	H1+SR	1/3H1+2/3SR	2/3H1+1/3SR	Mean	*SD*
S1: H1 only	1					16.11	3.40
S2: SR only	0.759	1				16.01	2.52
S3:1/2H1+1/2SR	0.956	0.917	1			32.12	5.56
S4:1/3H1+2/3SR	0.911	0.960	0.992	1		16.04	2.64
S5:2/3H1+1/3SR	0.983	0.866	0.994	0.971	1	16.07	2.96

Reliability

Reliability was calculated for the five different scoring scenarios for each form. The reliability for H1 was the lowest, ranging from .85–.89 across all the speaking items, the reliability for SR only was the highest, ranging from .94–.96; the other combinations had reliabilities above .89 for all the speaking items, see Table 2 below.

Table 2:
Reliability Range of Different Scoring Scenarios across Ten Forms

Scenario	Mean	*SD*	Reliability
H1 only	2.47-2.90	0.64-0.72	0.85-0.89
SR only	2.45-2.76	0.41-0.52	0.94-0.96
H1+SR	4.92-5.64	0.90-1.17	0.93-0.94
1/3H1+2/3SR	2.46-2.80	0.42-0.55	0.93-0.96
2/3H1+1/3SR	2.46-2.85	0.50-0.63	0.89-0.93

Reliability was calculated for the five different scoring scenarios for the combined 10 forms (see Table 3). The reliability for H1 was the lowest, .87, the reliability for SR only was the highest, .95; the reliability for other combinations were above .91 for all the speaking items. All of the reliabilities are within an acceptable range. It should be noted that combining a higher reliability score with one of lower reliability will typically result in a combined score having reliability somewhere between the higher and lower ones.

Table 3:
Reliability of Different Scoring Scenarios for Ten Forms

Scores	No. Items	Mean	*SD*	Cronbach's Alpha
H1 only	6	2.72	0.71	0.87
SR only	6	2.63	0.48	0.95
H1+SR	6	5.35	1.06	0.93
1/3H1+2/3SR	6	2.66	0.49	0.95
2/3H1+1/3SR	6	2.69	0.58	0.91

One-factor measurement model testing.

As was mentioned earlier, a single factor measurement model was fit to the data of each of the six scenarios. None of the three measurement models (congeneric, tau–equivalent, & parallel, see definition in Appendix B) fit the data well (see Table 4). The root mean square error approximation (RMSEA) was found to be in the range of .67 to .82, and the NFI and CFI were in the range of .53 to .67, all of which failed to meet the good model fit criteria. A single-factor measurement model that was proposed cannot be confirmed, which indicated that the different scenarios such as SpeechRater^{SM} only, Human Raters only, or the different combinations of the two cannot be considered as measuring the same ability dimension. This might be due to the relatively low correlation between SpeechRater^{SM} and human raters (.759) because this correlation is related to the regression used to derive the SpeechRater^{SM} model. The overall correlations between human rater 1 and 2 were .706. The case count of human rater 2 results was only 10% of the human rater 1 case count. The overall human rater 1, human rater 2 and SpeechRater^{SM} score comparison box plot can be found in Appendix A.

Table 4:

Results of Congeneric, Tau-equivalent and Parallel Hypothesis Testing

Factor Model	*N*	NFI	CFI	RMSEA
Congeneric	10,246	0.67	0.67	0.82
Tau-equivalent	10,246	0.65	0.65	0.67
Parallel	10,246	0.53	0.53	0.68

Research question 2

Do the scores obtained from SpeechRater^{SM} only, Human Raters only, or the different combinations of the two result in similar IRT parameter estimates (item difficulty and discrimination) and test takers' ability estimates of the population values and classifications?

Parameter analyses

Item difficulty.

In terms of item difficulty parameter differences for the six items (1-6), we found similar pattern among the six speaking items for the five scoring scenarios (S1 – S5) except for S2 (see Figure 1). S5 had the lowest estimated item difficulty parameters across the six items, followed by S1. There were not many differences when comparing S3 and S4. Item 2 had the lowest estimated item difficulty parameter values while item 6 had the highest item difficulty parameter estimates, but the differences between them were relatively small. S2 was close to S1 for items 1 and 4 and was very different for the other 4 items.

Item discrimination.

In terms of estimated item discrimination parameter differences for the six items (1-6), we found similar patterns across the five scenarios (see Figure 2). S1 and S5 had the lowest discrimination values, followed by S3, S4, and S2. Item 1 and 2 had the lowest discrimination values and other items had slightly higher values.

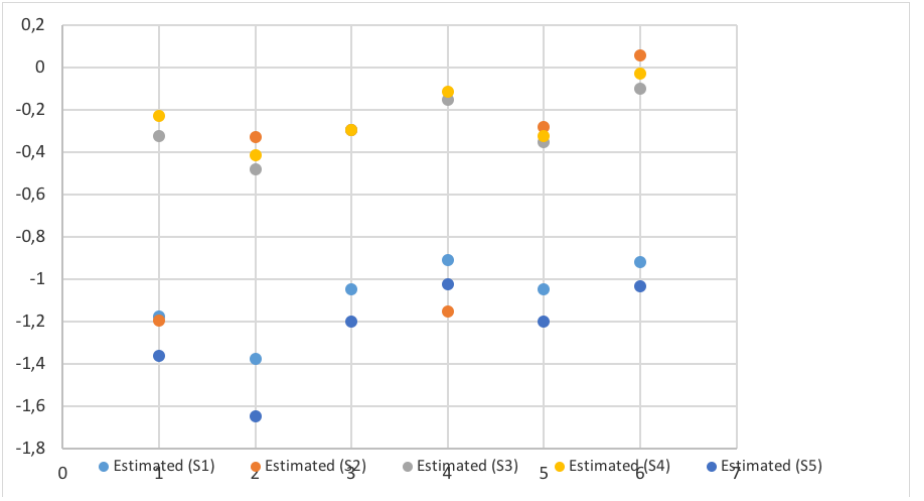


Figure 1:

Comparison of estimated difficulty parameters across five different scoring scenarios for the six speaking items.

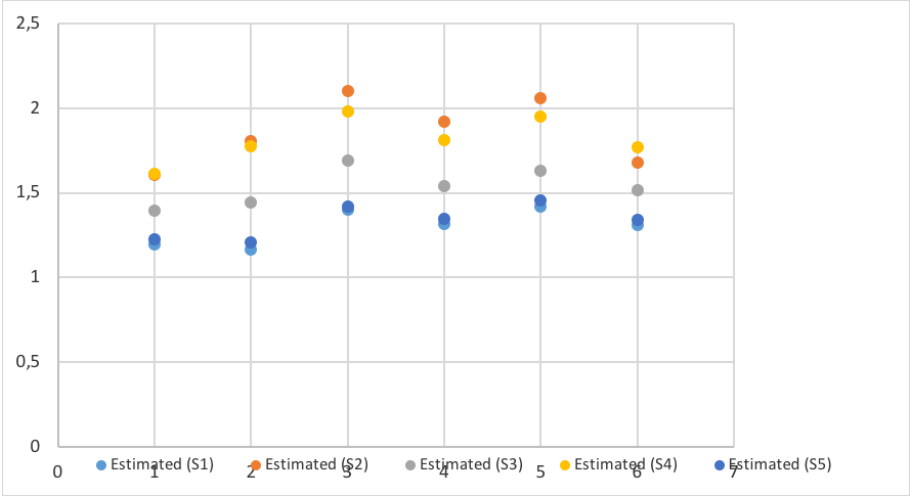


Figure 2:

Comparison of estimated discrimination parameters across five different scoring scenarios for the six speaking items.

Bias analyses

Ability estimates

In terms of test takers' estimated ability differences (see Figure 3), S1 and S2 had the largest absolute bias, RMSE and bias when combining all six items, followed by S1 and S4, S1 and S3, and S1 and S5.

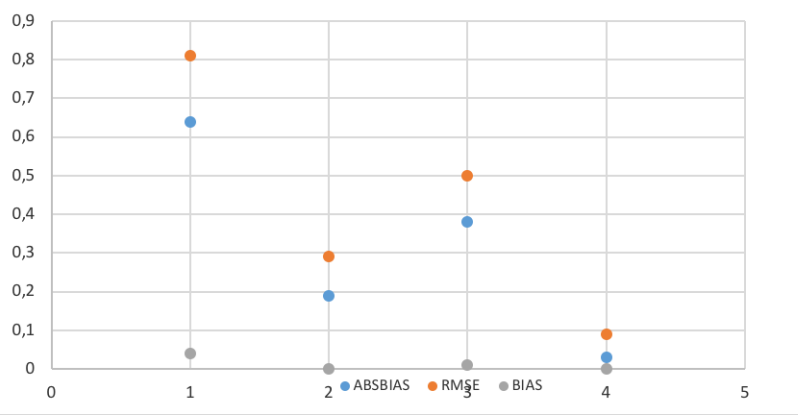


Figure 3:

Comparison of test takers' estimated ability values: comparison of four scoring scenarios against human rater (S1):1=S2- S1; 2=S3-S1; 3=S4-S1; 4=S5-S1.

Location estimates.

In terms of test takers' estimated location differences, S1 and S5 had the smallest absolute bias, RMSE and bias when combining all six items, followed by S1 and S2, S1 and S3, and S1 and S4 (Figure 4).

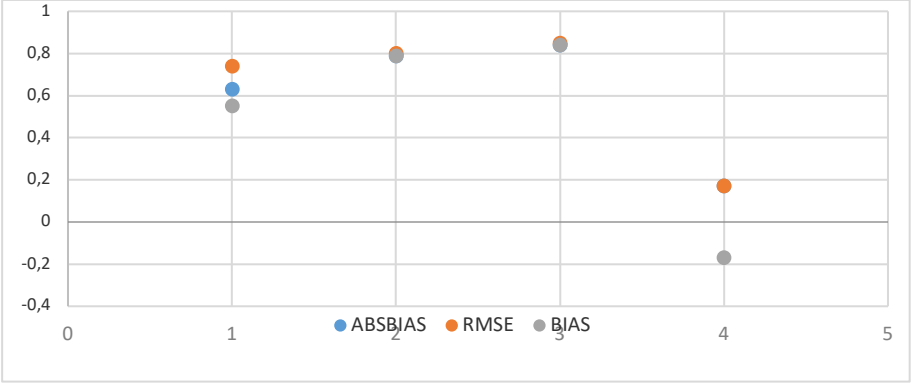


Figure 4:

Comparison of estimated location values: comparison of four scoring scenarios against human rater (S1):1=S2-S1; 2=S3-S1; 3=S4-S1; 4=S5-S1.

Slope estimates.

In terms of test takers' estimated slope differences, S1 and S2 had the largest absolute bias, RMSE and bias when combining all six items, followed by S1 and S4, S1 and S3, and S1 and S5 (Figure 5).

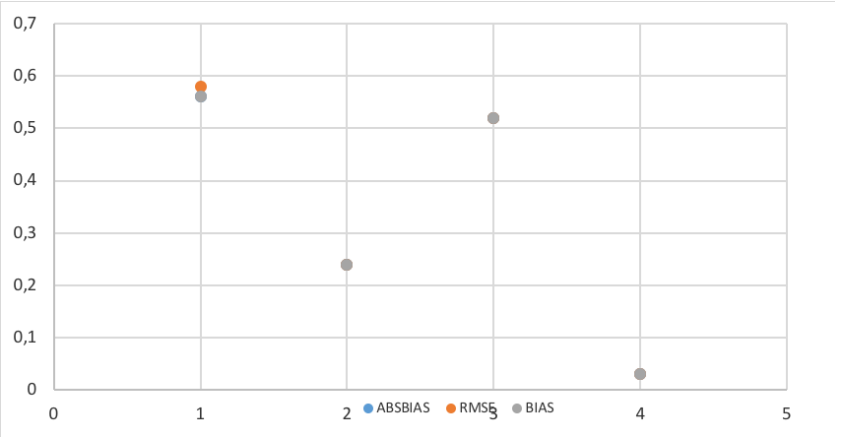


Figure 5:

Comparison of estimated slope values: comparison of four scoring scenarios against human rater (S1): 1=S2-S1; 2=S3-S1; 3=S4-S1; 4=S5-S1.

Classification

We used 3 hypothetical estimated ability values (theta) cut scores (0.0, 0.6 and 1.6) to classify test takers. We found that S5 was close to S1 at all the cut scores, followed by S3, S4 and S2 (Table 5).

Table 5:

Comparison of Test Takers' Passing Rate across Different Scoring Scenarios

	Theta=0.0	Diff vs S1	Theta=0.6	Diff vs S1	Theta=1.6	Diff vs S1
S1	46.06	–	32.19	–	5.43	–
S2	53.56	7.5	40.61	8.42	4.64	-0.79
S3	47.79	1.73	33.79	1.60	4.40	-1.03
S4	52.84	6.78	37.94	5.75	3.87	-1.56
S5	45.76	-0.3	32.19	0.00	6.19	0.76

Summary

In this paper, we summarized some findings regarding whether the scores obtained from SpeechRater[SM] only, Human Raters only, or the different combinations of the two result in similar IRT parameters (item difficulty and discrimination) and test takers' ability estimate and classifications. Below, we summarize the findings from our research and suggest recommendations for operational practice.

For Research Question 1, we found that although the overall correlations between H1-rater scores and SpeechRater[SM] scores were the lowest among all the different scoring designs, H1-rater had very high correlations with S5 and S3. The reliability for H1 was the lowest while the reliability for SR only was the highest (as would be expected because SR uses exactly the same criteria for scoring each time). A single-factor measurement model in the structural equation framework results indicated that all the different scoring scenarios such as SpeechRater[SM] only, Human Raters only, or the different combinations of the two cannot be considered as measuring the same ability dimension.

For Research Question 2, in terms of test takers' estimated ability differences, S1 and S5 had the smallest absolute bias, RMSE and bias when combining all six items. For the classification passing rate based on IRT ability estimates, the closest to H1-rater were S5 and S3. Speechrater[SM] yielded different results when compared with H1-rater at 2 out of 3 cuts. We also investigated the raw score distribution of both H1-rater and Speechrater[SM] scores and found that their distributions were different to some extent (see Figure 6 in the Appendix). The fact that the score distributions differ would mean that different cut scores might result in different findings with respect to the classifications. The test information curves and standard error curves of the Speechrater[SM] (S2) did not look similar to that of human rater (S1) (see Figures in Appendix C). Both test information curves and standard error curves indicated that S5 and S3 are closer to S1 than other scoring scenarios.

Generally speaking, some of the main findings indicate that the SR-only approach is most different from the H1-only approach in comparison to the other approaches (for example, it has the lowest correlation of 0.759 and a completely different pattern of difficulty parameter estimates across the six items, as shown in Figure 1). The explanation for these results would seem to be the fact that S3 - S6 all contain H1 in them (in combination with the SpeechRater[SM] score), so it is a given that they will be more similar to S1 than S2.

Discussion

In this study, we used different approaches, such as classical and IRT modeling techniques to compare human rater and Speechrater[SM] scores. We created different scoring scenarios based on arbitrary weights and investigated their differences. It seems that these approaches are effective in detecting the differences between human and automated scoring; the research results can help practitioners make decisions in terms of how to combine human and machine scores in the field. Identifying issues and differences between Speechrater[SM] and human rater can help improve Speechrater[SM]. However, the real data used for the human rater might not be perfect (Wang & Yao, 2013), which may have

prevented us from finding the real issue. Simulation studies are needed to compare Speechrater[SM] and human rater in a controlled manner.

In this study we used real data and investigated the differences between different scoring methods. We found that there are some systematic patterns in the combined scenarios based on both classical and IRT approaches, such as their raw score distributions, test information curves (see Appendix C), standard error curves (see Appendix C), and percentage of passing rate. As pointed out above the percentage of passing rate results may well differ for different cut scores. The fact that S5 and S3 are closer to S1 than other scoring scenarios is within our expectation because they account for 1/2 and 2/3 of the human score.

In this study, we compared the differences among different combinations of human and machine scores, and more such studies are needed (Attali, 2013; Zhang et al., 2013). We believe that the use of the statistical analyses (both classical and IRT) in this study is a useful way to advance the study of automated scoring in the evaluation of speech responses. Our study can help our clients make decisions related to machine and human scoring. A future study is needed to provide guidelines about how to establish the set of weights that will generate optimal reliability to combine human rater and SpeechRater[SM] scores. Reliability is just one criteria and there might be other criteria, such as validity that are also important to investigate. A simulation study should be conducted to test different scoring combination scenarios, and different cut scores, under different conditions.

Limitations

Since human raters' scores are based on three areas (delivery, language use, and topic development) while SpeechRater[SM] scores are based only on delivery and language use, differences between the two types of scores will probably always exist (and these differences relate to validity issues). There are issues and questions that need to be investigated and answered before an automated speech scoring system such as SpeechRater[SM] can be implemented for the operational scoring of English language assessment speaking items.

We want to stress that the automated speech scoring engine SpeechRater[SM] is still evolving, and that additional features, covering a more extended subset of the Speaking construct, are currently under development and are planned for use in future scoring models. Moreover, we are also exploring alternative approaches to the standard linear regression scoring model currently used by SpeechRater[SM], which may lead to improvements in its scoring performance. The current Speechrater[SM] distribution has high frequency at middle scores (see Figure 6 in the Appendix C), which may be due to the regression toward the mean effects from the linear regression scoring model.

Notes

1–3. See definitions in Appendix B.

References

Attali, Y. (2013). Validity and reliability of automated essay scoring. In M. D. Shermis & J. Burstein (Eds.) *Handbook of automated essay evaluation: Current applications and new directions* (pp. 181-198). New York, NY: Routledge.

Attali, Y., Bridgeman, B., & Trapani, C. (2010). Performance of a generic approach in automated scoring. *Journal of Technology, Learning, and Assessment*, *10*, 1-16. Retrieved from http://www.jtla.org.

Attali, Y., & Burstein, J. (2006). Automated essay scoring with e–rater®v.2. *Journal of Technology, Learning, and Assessment, 4*, 1-30. Retrieved from http://www.jtla.org.

Bennett, R. E., & Bejar, I. I. (1998). Validity and automated scoring: It's not only the scoring. *Educational Measurement: Issues and Practice, 17*, 9–17.

Browne, M. W., & Cudeck, R. (1993). Alternative ways of assessing model fit. In K. A. Bollen & J. S. Long (Eds.). *Testing structural equation models* (pp. 136–162). Beverly Hills, CA: Sage.

Chen, M., & Zechner, K. (2011). *Computing and evaluating syntactic complexity features for automated scoring of spontaneous non–native speech.* Proceedings of the 49th Annual Meeting of the Association for Computational Linguistics and the Human Language Technologies Conference (ACL–HLT–2011) (pp.722-731). Portland, OR: Association for Computational Linguistics.

Chodorow, M., & Burstein, J. (2004). *Beyond essay length: Evaluating e–rater®'s performance on essays* (Research Report No. RR–04–04). Princeton, NJ: Educational Testing Service.

Higgins, D., Xi, X., Zechner, K., & Williamson, D. M. (2011). A three-stage approach to the automated scoring of spontaneous spoken responses. *Computer Speech and Language, 25*, 282-306.

Hooper, D., Coughlan, J., & Mullen, M. R. (2008). Structural equation modeling: Guidelines for determining model fit. *Electronic Journal of Business Research Methods, 6*, 3–60.

Hu, L., & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criterion versus new alternatives. *Structural Equation Modeling, 6*, 1–55.

Jöreskog, K., & Sörbom, D. (1984). *LISREL IV: Analyses of linear structural relationships by maximum likelihood, instrumental variables, and least squares methods*. Mooresville, IN: Scientific Software.

Laundauer, T. K., Laham, D. & Foltz, P. (2003). Automatic essay assessment. *Assessment in Education, 10*, 295–308.

Maruyama, G. (1998). *Basics of structural equation modeling.* Thousand Oaks CA: Sage.

Nichols, P. (2005). Evidence for the interpretation and use of scores from an automated essay scorer (White Paper). Iowa City, IA: Pearson.

Ramineni, C., Trapani, C., Williamson, D. M., Davey, T., & Bridgeman, B. (2012). *Evaluation of e-rater® for the GRE issue and argument prompts* (Research Report No. RR–12–06). Princeton, NJ: Educational Testing Service.

Streeter, L., Bernstein, J., Foltz, P., & Deland, D. (2011). *Pearson's automated scoring of writing, speaking, and mathematics* (White Paper). Iowa City, IA: Pearson.

Wang, Z., & von Davier, A. A. (2014). *Monitoring of scoring using the e-rater automated scoring system and human raters on a writing test* (Research Report No. RR–14–04). Princeton, NJ: Educational Testing Service.

Wang, Z., & Yao, L. (2013). *Investigation of the effects of scoring designs and rater severity on students' ability estimation using different rater models* (ETS Research Report. No. RR–13–23). Princeton, NJ: Educational Testing Service.

Wang, Z., Zechner, K., & Sun, Y. (2016). Monitoring the performance of human and automated scores for spoken responses. *Language Testing*. Advance online publication. doi: 10.1177/0265532216679451

Williamson, D. M., Xi, X., & Breyer, F. J. (2012). A framework for evaluation and use of automated scoring. *Educational Measurement: Issues and Practice*, *31*(1), 2–13.

Xie, S., Evanini, K., & Zechner, K. (2012). Exploring content features for automated speech scoring. *Proceedings of North American Chapter of the Association for Computational Linguistics – Human Language Technologies (NAACL–HLT),* Montreal, Canada: Association for Computational Linguistics.

Yoon, S. Y., & Bhat, S. (2012). *Assessment of ESL learners' syntactic competence based on similarity measures.* Paper presented at the Joint Conference on Empirical Methods in Natural Language Processing and Computational Natural Language Learning (EMNLP-CoNLL) (pp. 600–608). Stroudsburg, PA: Association for Computational Linguistics.

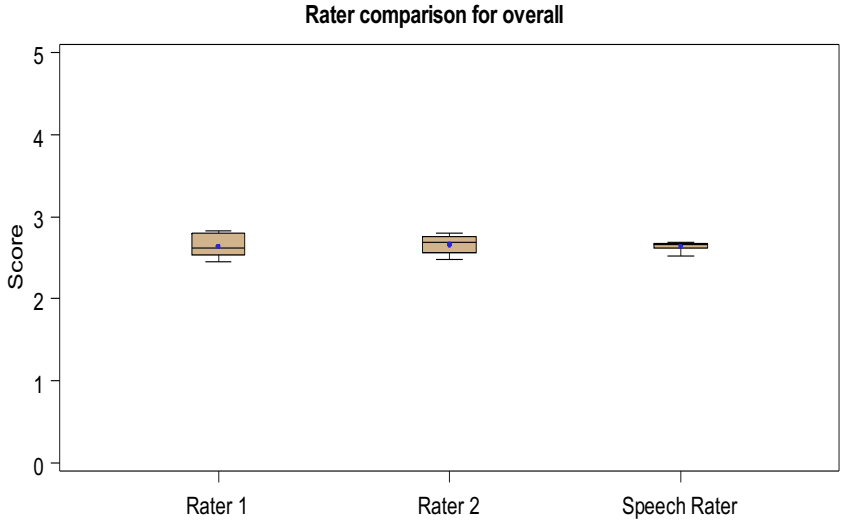
Yoon, S.-Y., Bhat, S., & Zechner, K. (2012). *Vocabulary profile as a measure of vocabulary sophistication.* Paper presented at the 7th Workshop on Innovative Use of NLP for Building Educational Applications, North American Chapter of the Association for Computational Linguistics – Human Language Technologies (NAACL–HLT). Montreal, Canada: Association for Computational Linguistics.

Zechner, K., Higgins, D., Xi, X., & Williamson, D. M. (2009). Automatic scoring of non-native spontaneous speech in tests of spoken English. Special issue on Spoken Language Technology for Education. *Speech Communication, 51*, 883–895.

Zhang, M. (2013). *Contrasting automated and human scoring of essays*. (R & D Connections, No. 21). Princeton, NJ: Educational Testing Service.

Zhang, M., Breyer, J., & Lorenz, F. (2013). *Investigating the suitability of implementing e-rater in a large-scale English language testing program.* (Research Report RR-13–36). Princeton, NJ: Educational Testing Service.

Appendix A



Note. Human rater 2 is only 10% of the H1 data.

Figure A1. Rater comparison between rater 1, rater 2 and SpeechRater[SM].

Appendix B

Parallel measures

The measures (items) comprising a scale are parallel if the following two conditions hold:

1. $\tau_{ip} = \tau_{jp}$ for all *i* and *j*; τ = true scores; *p* = person;
2. $\sigma^2_{ei} = \sigma^2_{ej}$ for all *i* and *j*: e = error:

This implies that the amount of variation in the item score that is determined by the true score is the same for all items. It also implies that the expected value of each of the items will be the same.

Tau-equivalent measures

When measures are tau-equivalent, $\tau_{ip} = \tau_{jp}$ for all *i* and *j*, as in the case of parallel measures, but we relax the assumption that $\sigma^2_{ei} = \sigma^2_{ej}$ for all *i* and *j*.

Congeneric measures

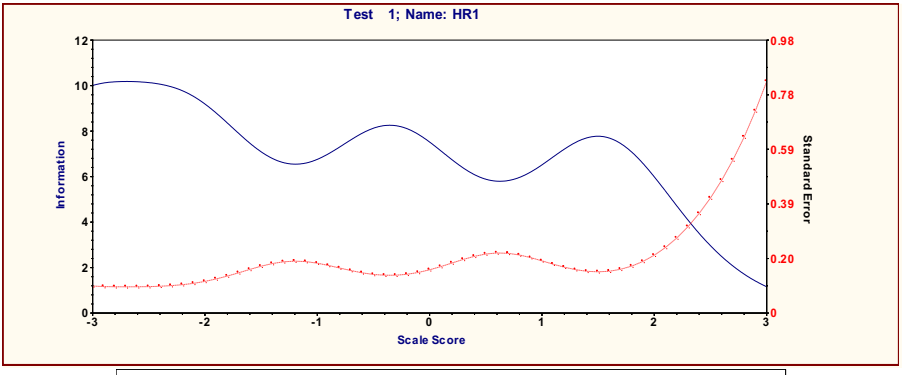
Congeneric measures relax both the assumption that $\tau_{ip} = \tau_{jp}$ for all i and j, and that $\sigma_{ei}^2 = \sigma_{ej}^2$ for all i and j.

Appendix C

Test Information Curves and Standard Error Curves

Figure C1:

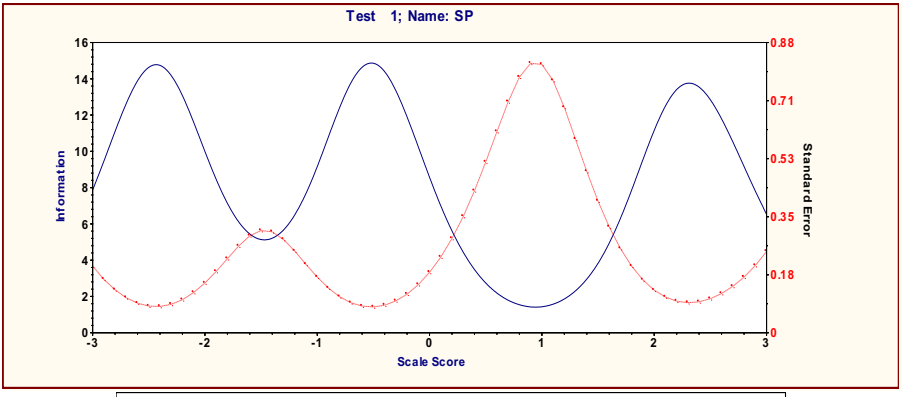
S1.



Test information curve: solid line Standard error curve: dotted line
The total test information for a specific scale score is read from the left vertical axis.
The standard error for a specific scale score is read from the right vertical axis.

Figure C2:

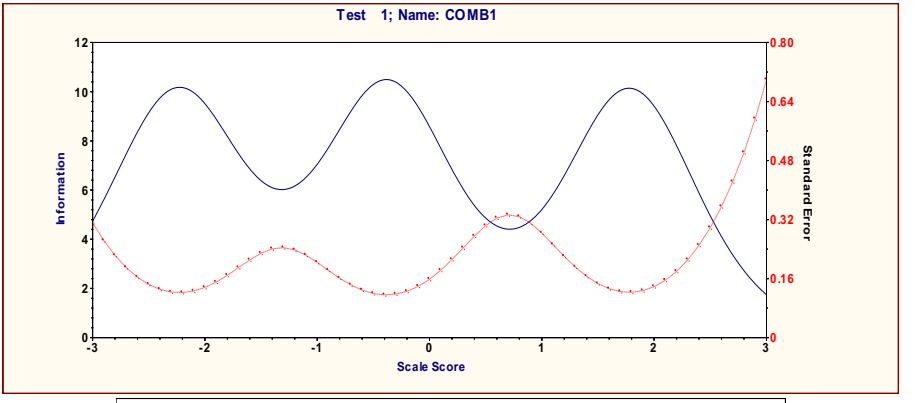
S2



Test information curve: solid line Standard error curve: dotted line
The total test information for a specific scale score is read from the left vertical axis.
The standard error for a specific scale score is read from the right vertical axis.

Figure C3:

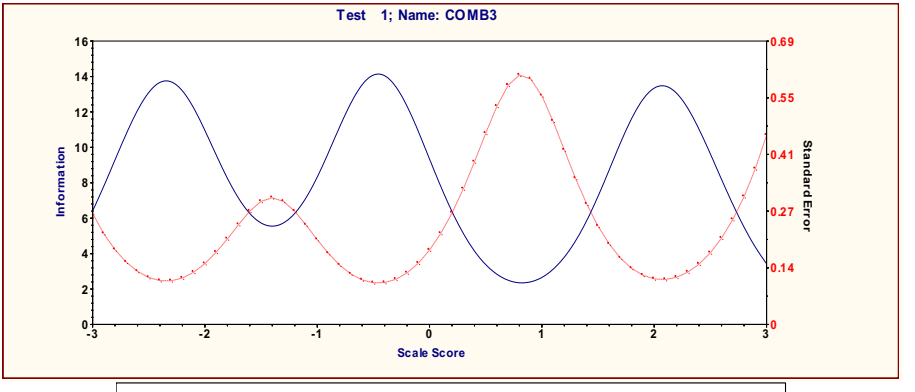
S3.



Test information curve: solid line **Standard error curve: dotted line**
The total test information for a specific scale score is read from the left vertical axis.
The standard error for a specific scale score is read from the right vertical axis.

Figure C4:

S4



Test information curve: solid line **Standard error curve: dotted line**
The total test information for a specific scale score is read from the left vertical axis.
The standard error for a specific scale score is read from the right vertical axis.

Figure C5:

S5

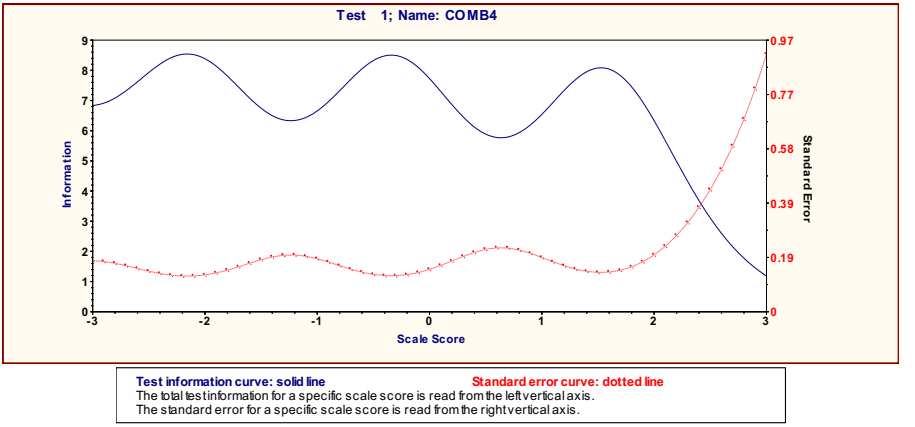
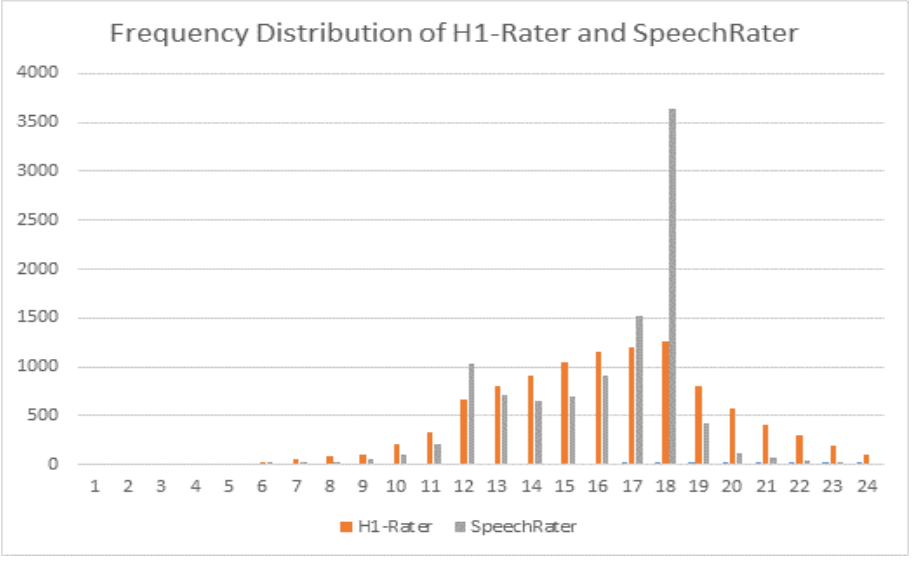


Figure C6:

Frequency distribution of H1-Rater and SpeechRater[SM].



Psychological Test and Assessment Modeling, Volume 60, 2018 (1), 101-139

Item response models for human ratings: Overview, estimation methods, and implementation in R

Alexander Robitzsch[1] *& Jan Steinfeld*[2]

Abstract

Item response theory (IRT) models for human ratings aim to represent item and rater characteristics by item and rater parameters. First, an overview of different IRT models (many-facet rater models, covariance structure models, and hierarchical rater models) is presented. Next, different estimation methods and their implementation in R software are discussed. Furthermore, suggestions on how to choose an appropriate rater model are made. Finally, the application of several rater models in R is illustrated by a sample dataset.

Keywords: multiple ratings, many-facet rater model, hierarchical rater model, R packages, parameter estimation, item response models

[1]Leibniz Institute for Science and Mathematics Education (IPN) at Kiel University, Kiel, Germany, and Centre for International Student Assessment, Germany. *Correspondence concerning this article should be addressed to:* Alexander Robitzsch, PhD, IPN, Olshausenstraße 62, D-24118 Kiel, Germany; email: robitzsch@ipn.uni-kiel.de

[2]Federal Ministry of Education, Science and Research, Austria

1 Introduction

Educational assessments often involve different approaches and procedures. Some abilities can be measured with closed answering formats such as multiple-choice questions, while other competencies, for example, expressive (productive) competencies, require constructed-response formats. One reason for why the latter are not so commonly used in large-scale assessments is that these kinds of tasks mostly require human judgment (rather than computer programs) to score answers or to assess their quality. Besides educational and language assessment, many other areas of testing require human judgment as well, such as the scoring of students within medical education programs (Tor & Steketee, 2011), the assessment of abilities using the approach of multiple mini-interviews (McLaughlin, Singer, & Cox, 2017), or large-scale placement tests (S. M. Wu & Tan, 2016). Therefore, possible rater effects must be taken into consideration.

Wind and Peterson (2018), who conducted a systematic review of the methods used in different application areas of rater studies, found that the research focus varies greatly. Some studies focus on the estimation of item difficulties, while others are more interested in the rating quality or the estimation of test-takers' ability. It is important to consider the main purpose of each study and to take into account the fact that the research focus may result in different study designs and that some estimation methods are superior to others. The research design and the estimation method chosen depend on the research question being investigated. Furthermore, the question of what kind of role the items and persons should have in the specific research should be considered.

First, items and persons could be seen as fixed, which means that each item and each person is associated with fixed model parameters, namely, an item difficulty and a person ability. As a consequence, the item responses X_{pi} of person p to item i are modeled as $P \times I$ independent random variables, given the fixed model parameters. Second, if persons and/or items are treated as random, this means that either persons and/or items are a random sample, it is necessary to make assumptions about the underlying distributions (see De Boeck, 2008). The consequences of the persons and/or items being treated as either fixed or random are that there is a change in the interpretation of the parameters and the resulting probabilities in the item response models (IRT models).

Performances are often graded by multiple raters in order to increase the reliability and objectivity of the ratings and to minimize rater errors (see Eckes, 2015 for a comprehensive overview). The expected degree of agreement (or nonagreement) depends on the attitudes and expectations of the raters, their knowledge, and the study design applied. For example, in studies in which raters grade performance more holistically (e.g., if there are no specific guidelines on how the raters should score the performance), a lower agreement is expected. When detailed scoring rules are applied, higher rater agreement can be expected. If detailed scoring rules are applied and broad training is provided for the raters, the ratings can be expected to be more homogeneous in terms of higher agreement between the performance scores. However, it could be expected that the application of detailed scoring rules yields ratings that are no longer locally independent, which is

a typical assumption made in IRT models (the residuals might correlate substantially; Wang, Su, & Qiu, 2014, see also Verhelst & Verstralen, 2001). The variable behavior of raters can be summarized under the label "rater effects". Depending on the knowledge of the raters, their attitudes, and their expectations of the performance, different raters may give different grades. Well-known rater effects include the effect of severity/leniency (Engelhard, 1992; Lunz, Wright, & Linacre, 1990), the halo effect (Bechger, Maris, & Hsiao, 2010; Myford & Wolfe, 2003), the central tendency, and the restriction of range of judgments (Engelhard, 1994; Saal, Downey, & Lahey, 1980).

Many different statistical approaches for analyzing multiple ratings are discussed in the literature (Eckes, 2017). To begin with, generalizability theory (G-theory; Brennan, 2001a) decomposes the total variance on a raw score metric (scores of raters on performance) into the additive variance components of the person, the items, and the raters. Both double and triple interactions (persons $\times$ items, persons $\times$ raters, and persons $\times$ items $\times$ raters) can be considered. G-theory treats items and persons as a sample of a theoretically infinite population of items and persons. The G-theory is useful regarding, for example, the formulation of rater effects, but it is also limited as the relationship of the components is treated as linear and additive in the raw score metric of items, which might not be appropriate.

In the context of the item response theory (IRT), several other methods have been proposed to model rater effects. These approaches are mostly based on the concept of virtual items, which are defined as the set of all combinations of original items and raters (see Rost & Langeheine, 1997). For example, in the case of two items and four raters, $2 \times 4 = 8$ virtual items can be created. A virtual item for a particular original item and a particular rater includes all ratings of the corresponding original item and rater, respectively. Based on virtual items, in the many-facet Rasch model (Linacre, 1989, 2017), the ratings of raters on all items and on all persons are decomposed into the additive effects of persons, items, and raters on the logit metric (more precisely, item $\times$ rater, or a matrix in which student essay $\times$ rater is shown). As illustrated in Figure 1a, each of the four raters rates two items. In total, there are two items and the responses to each of these two items are partitioned into four virtual items. The residuals among the virtual items are treated as being locally stochastically independent given a general person ability variable. A typical example of Figure 1a is the many-facet Rasch model, which results from the application of a restricted partial credit model to virtual items. Systematic differences in rater behavior are modeled by allowing item difficulties to differ between raters. However, the ratings that correspond to generalized items are assumed to be locally stochastically independent. This assumption is typically violated in many applications because the ratings of one single item by two raters will appear to be more similar than the ratings of two different items by two raters. Therefore, additional person-item interaction effects have to be considered.

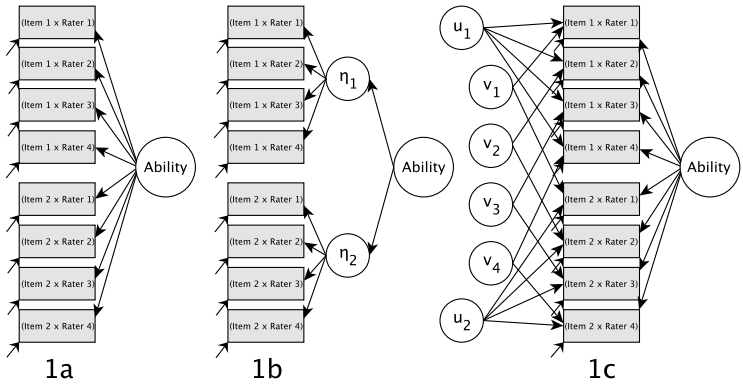


Figure 1:

All models 1a, 1b, and 1c represent eight virtual items, where each rater rated two of the virtual items. In both models 1a and 1b, the ratings were locally independent, whereas in model 1c, the additional parameters u and v were introduced to account for the interaction between persons and items as well as between persons and raters. Model 1a depicts the many-facet rater model, model 1b the hierarchical rater model, and model 1c the generalized many-facet rater model.

In Figure 1, the additional dependence caused by rating the same item is taken into account by a hierarchical rater model (Patz, Junker, Johnson, & Mariano, 2002; DeCarlo, 2005; DeCarlo, Kim, & Johnson, 2011). Person ability causes true ratings η of the two items, which are themselves measured by 2×4 observed ratings (i.e., the virtual items). Moreover, it is possible that the rating of a particular rater on the first item influences the rating on the second item (halo effect). In this case, additional dependence is introduced and the local independence assumption in Figure 1b is violated. In the generalized many-facet rater model depicted in Figure 1c, person-item and person-rater interactions are modeled by additional random effects (latent variables; Wang et al., 2014) that capture the violation of local independence in Figures 1a und 1b. It should be noted that local dependence can be alternatively represented as correlated residuals in Figure 1c. In the next section, these three different modeling approaches are formally described and are introduced as special cases of IRT models applied to the polytomous item responses of virtual items.

2 Item Response Models for human raters

In the following section, different item response models for human ratings are introduced. First, an overview of IRT models is presented. Then, these IRT models are extended to include rater effects for modeling rating data for human raters. In particular, we distinguish between the approaches of many-facet rater models, covariance structure models, and hierarchical rater models.

2.1 Item response models for polytomous data

Here, we provide a short review of the most frequently used IRT models for polytomous data. With X_{pi} we denote the polytomous item response of person p to item i. While the items are often treated as fixed, person parameters are often assumed to be random (see Holland, 1990) and are modeled by a distribution (e.g., a normal distribution or located latent classes). In the following description, we will mostly choose a unidimensional distribution of the ability (latent trait) θ_p, although the extension to multidimensional traits does not substantially change the interpretation of the models.

Partial credit model

The partial credit model (PCM; Masters, 1982) is an item response model for two or more ordered categories. The item response probability for responding to category $k = 0, \ldots, K_i$ is given as

$$P(X_{pi} = k|\theta_p) \propto \exp\{k\theta_p - b_{ik}\} \tag{1}$$

The symbol $\propto$ means that the right-hand side of Equation (1) sums to one across all categories k. The model has the property that persons with high abilities θ_p tend to respond in high categories k. The parameter b_{ik} indicates an item-category-specific intercept. This parameter is also often reparameterized in the form $b_{ik} = k\beta_i - \sum_{h=0}^{k} \tau_{ih}$ with a general item difficulty β_i and item thresholds τ_{ih}. The PCM belongs to the family of Rasch models and shares the important properties of the Rasch model that the sum score $S_p = \sum_i X_{pi}$ is a sufficient statistic for the person parameter θ_p and the person and item parameters are separable (Andersen, 1980). Therefore, conditional maximum likelihood estimation can be used as an estimation approach that provides item parameter estimates without the need to specify the ability distribution (see Section 3). A restricted form of the PCM is the linear logistic test model (LLTM; Fischer, 1973), which models the item-specific intercepts as a linear function of basis parameters and is given as

$$b_{ik} = \sum\nolimits_{m=1}^{M} q_{ikm}\gamma_m \tag{2}$$

where γ_m are basis item parameters and q_{ikm} are known prespecified values. Specific hypotheses can be tested by imposing restrictions on the PCM in Equation (1). For example, a rating scale model (Andrich, 1978) can be formulated as a particular LLTM, in which the model has item difficulty parameters and item thresholds that are assumed to be invariant across items.

Generalized partial credit model

The generalized partial credit model (GPCM) is a generalization of the PCM and was introduced by Muraki (1992). This model includes an additional item-specific discrimination parameter a_i and allows the items to have different reliabilities. It is formulated as

$$P(X_{pi} = k|\theta_p) \propto \exp\{ka_i\theta_p - b_{ik}\} \tag{3}$$

In most applications, the GPCM provides a better model fit than the PCM. Items with larger item discriminations are preferred because they are more informative in discriminating between persons with lower and higher ability values. As in the PCM, item discriminations a_i as well as item-category intercepts b_{ik} can be modeled as linear functions of the basis item parameters (Embretson, 1999); this makes the estimation of more parsimonious models possible. It should be emphasized that the weighted sum score $S_p = \sum_i a_i X_{pi}$ is a sufficient statistic for the person parameter θ_p.

Graded response model

The graded response model (GRM) proposed by Samejima (1969) belongs to the class of so-called cumulative IRT models. The item response probabilities are given as

$$P(X_{pi} = k|\theta_p) = G(a_i\theta_p - b_{i,k+1}) - G(a_i\theta_p - b_{ik}) \qquad (b_{i0} = 0, b_{i,K_i+1} = \infty) \quad (4)$$

where G is a link function that is typically the logistic link function or the probit link function. The model includes item discriminations a_i and ordered item intercepts b_{ik}. It is often found that the GRM and the GPCM provide similar fit to empirical datasets (Forero & Maydeu-Olivares, 2009) and, hence, there are no crucial consequences of choosing one of the two models. Again, the item parameters can be formulated as linear functions to estimate restricted versions of the GRM. For the probit link function, Equation (4) can be rewritten as $X^*_{pi} = a_i\theta_p + e_{pi}$ where X^*_{pi} is an underlying continuous variable for the ordinal item X_{pi} and e_{pi} is a standard, normally distributed residual. The ordinal item X_{pi} is obtained by discretizing the continuous variable X^*_{pi} with respect to thresholds b_{ik}. Using the variable X^*_{pi} has the advantage that correlated residuals can be specified in the GRM, which can model violations of local independence. However, in this situation, marginal maximum likelihood estimation is no longer computationally feasible and limited information estimation procedures have to be applied (see Section 3).

Covariance structure model

The normal distribution is probably the most frequently applied distribution. Sometimes the question arises whether the normal distribution can also be applied to ordinal items. However, the probability density of the normal distribution is defined on the real line and not on discrete values. Therefore, a misspecified model results if the normal distribution is applied to ordinal items. The assumed normal density is given as

$$f(X_{pi} = k|\theta_p) = (2\pi\sigma_i)^{-\frac{1}{2}} \exp\Big\{ - \frac{(k - a_i\theta_p - \mu_i)^2}{2\sigma_i^2} \Big\}, \text{ i.e. } X_{pi} = \mu_i + a_i\theta_p + e_{pi} \quad (5)$$

An item i is parameterized with an item mean μ_i, an item discrimination a_i, and a residual variance $\sigma_i^2 = Var(e_{pi})$. Unfortunately, the item parameters of the GPCM or the GRM cannot be simply converted into the parameters of the normal distribution in Equation (5). However, in some applications, the item and distribution parameters

from the covariance structure model (CSM; often referred to as confirmatory factor analysis) shown in Equation (5) can be more easily interpreted than the parameters of the GPCM or GRM. More formally, in a CSM, the mean vector $\boldsymbol{\mu} = \boldsymbol{\mu}(\boldsymbol{\gamma})$ of the I items $X_{p1}, \ldots, X_{pI}$ and the covariance matrix $\boldsymbol{\Sigma} = \boldsymbol{\Sigma}(\boldsymbol{\gamma})$ are modeled as functions of an unknown parameter vector $\boldsymbol{\gamma}$ (Bollen, 1989). In a CSM, the covariance matrix is represented as $\boldsymbol{\Sigma} = \boldsymbol{\Lambda}\boldsymbol{\Phi}\boldsymbol{\Lambda}^T + \boldsymbol{\Psi}$, where $\boldsymbol{\Lambda}$ is the loading matrix, $\boldsymbol{\Phi}$ is the factor covariance matrix and $\boldsymbol{\Psi}$ is the residual covariance matrix. Then, the vector $\boldsymbol{\gamma}$ contains elements of the mean vector, loadings and elements of the factor covariance, and residual covariance matrices. When applied to ordinal data, the CSM is a so-called pseudo-likelihood estimation approach as the assumed likelihood function is misspecified (Yuan & Schuster, 2013). Interestingly, Arminger and Schoenberg (1989) showed that the mean structure and the covariance structure in Equation (5) can be consistently estimated in a confirmatory factor analysis based on a misspecified normal distribution for ordinal items. However, so-called maximum likelihood robust standard errors should be used, in order to ensure that valid statistical inferences can be made in the case of a misspecified likelihood (White, 1982). Alternatively, the bootstrap resampling method of persons can be used to obtain valid standard errors (Berk et al., 2014).

2.2 Many-facet rater model

The IRT models presented in the following paragraphs are based on virtual items of every combination of an item and a rater (see Figure 1). We denote the corresponding item responses as X_{pir} for person p to item i rated by rater r. Unidimensional many-facet rater models (MFRM) are obtained by applying the PCM, the GPCM, or the GRM to these virtual items. The item response probability in the extension of the GPCM is given as

$$P(X_{pir} = k|\theta_p) \propto \exp\{ka_{ir}\theta_p - b_{ikr}\} \tag{6}$$

and, for the GRM, it is written as

$$P(X_{pir} = k|\theta_p) = G(a_{ir}\theta_p - b_{ir,k+1}) - G(a_{ir}\theta_p - b_{irk}) \tag{7}$$

Typically, constrained versions of these models are applied to rating data. In the family of Rasch models, the item discriminations a_ir in the GPCM (Equation 6) are all set to one (also labeled as Rasch-MFRM in the following). A Rasch-MFRM (Linacre, 1989) imposes additional restrictions on item parameters such that

$$P(X_{pir} = k|\theta_p) \propto \exp\{k\theta_p - k\beta_i - k\alpha_r - \sum\nolimits_{h=0}^{k} \tau_{ih}\} \tag{8}$$

In this specification, the parameter β_i refers to the general item difficulty, α_r is the rater severity parameter and τ_{ih} are item-step parameters. The model specified in Equation (8) is a particular LLTM of the PCM applied to virtual items with linear constraints on item-category intercepts b_{irk}. It should be emphasized that rater effects are assumed to be homogeneous across all items in Equation (8). An important extension to Equation

(8) is the introduction of further interaction effects between items and raters, which allows for systematic item-specific rating behavior. The more restrictive model with homogeneous rater effects can be tested against the more complex model that allows for item-rater interactions. Rater centrality/extremity (see Wolfe, 2014) can be modeled by including rater-step parameters α_{rh} in Equation (8). We note that an identification condition has to be assumed in order to estimate (8) (e.g., $\Sigma_r \alpha_r = 0$).

The Rasch-MFRM has the advantage that the unweighted sum score $S_p = \Sigma_{ir} X_{pir}$ is a sufficient statistic for the person parameter θ_p. By using the many-facet Rasch model as a scaling model for obtaining person parameter estimates, an implicit decision about an equal weighting of items is made. From the perspective of item fit in real data applications, items as well as raters will typically assess the person ability with different precision. Therefore, an item response model that includes discrimination parameters will almost always result in better model fit. Besides severity-leniency effects or scale-usage effects of raters, raters can also differ in the reliability of the ratings they provide. The item-rater discrimination parameter $a_i r$ is a measure of the reliability of the ratings of item i and rater r (M. Wu, 2017). A more parsimonious model, which can also often be useful, linearly decomposes the item-rater discrimination, such that $a_{ir} = a_i + a_r$. Submodels that include only item discriminations ($a_{ir} = a_i$) or only rater discriminations ($a_{ir} = a_r$) provide further interesting diagnostic tools for studying the behavior of items and raters.

We emphasize that the GPCM (6) and the GRM (7) are often specified in a restricted form in which item-rater parameters follow a linear function, such as in the LLTM. These models are implemented in the R packages discussed in Section 3 of this paper.

2.3 Generalized many-facet rater model

As argued in the introduction, ratings are not locally independent across items and raters. First, different raters evaluate the performance of a student on an item, which typically introduces some dependency because additional item-specific factors besides general ability are at play. Hence, an additional student-item interaction effect has to be modeled. Second, the rating of one item by a rater can also influence the rating of another item by the same rater (the halo effect). Therefore, an additional student-rater interaction needs to be modeled. The MFRM can be extended to include these two additional random effect parameters u_{pi} and v_{pr} to model local dependence. The resulting generalized many-facet rater model (GMFRM; Wang et al., 2014; Verhelst & Verstralen, 2001, for a version for dichotomous ratings) can be written as

$$P(X_{pir} = k|\theta_p, u_{pi}, v_{pr}) \propto \exp\{k\alpha_i\theta_p + ku_{pi} + kv_{pr} - k\beta_i - k\alpha_r - \sum\nolimits_{h=0}^{k} \tau_{ih}\} \quad (9)$$

Several submodels of (9) can be estimated. A version of (9) that sets all item discriminations a_i to one is a multidimensional Rasch model (Wang et al., 2014) with random person-item and person-rater effects. The size of the variance components $\sigma_i^2 = Var(u_{pi})$ and $\sigma_r^2 = Var(v_{pr})$ quantifies the degree of the dependency of the

ratings. In some applications, it seems useful to include only the item or rater random effect for local dependence. The GMFRM models the additional dependence caused by ratings of the same items and by the same raters as additional random effects that prevent the assumption of local independence. Alternatively, the GPCM (9) can be substituted by a GRM using a latent variable representation. Using this approach, the random effects can be integrated out so that only person ability appears as a person variable in the model (Tuerlinckx & De Boeck, 2004; see also Ip, 2010). However, this equivalent model introduces correlated residuals, as rating variables X_{pir} for the same item i and for the same rater r are typically positively correlated. It must be emphasized that moving from the model with random effects to the equivalent model with correlated residuals implies a change in the metric of item parameters because the ability metric has changed. More formally, integrating out the random effects u_{pi} and v_{pr} from (9) results in the conditional response probability

$$P(X_{pir}=k|\theta_p) \propto \exp\{k\lambda_{ir}\alpha_i\theta_p - k\lambda_{ir}\beta_i - k\lambda_{ir}\alpha_r - \sum\nolimits_{h=0}^{k}\lambda_{ir}\tau_{ih} \qquad (10)$$
$$\text{with} \quad \lambda_{ir} = (\delta^2\sigma_i^2 + \delta^2\alpha_r^2 + 1)^{\frac{1}{2}}\}$$

where $\delta = 0.583$ is a positive constant (see Ip, 2010). As the multiplication factor λ_{ir} is always smaller than one, all item parameters are shrunken to the extent of local dependence caused by person-item and person-rater interactions. Hence, comparisons of the item parameters of the GMFRM and the MFRM should consider the transformation formula in Equation (10) for item parameters. The size of the residual correlations in (10) can also be computed based on the variance components of the random effects in model (9).

2.4 Covariance structure model and generalizability theory

Instead of modeling the ordinal virtual items of the rating data with an item response model for polytomous item responses, a CSM can alternatively be applied using normal distributions for modeling the virtual items X_{pir}. The mean structure can be represented by general item effects and general rater effects for modeling severity. The covariance structure can be modeled as a confirmatory factor model $\mathbf{\Sigma} = \mathbf{\Lambda\Phi\Lambda}^T + \mathbf{\Psi}$ in which the distribution parameters of person ability are represented in the covariance matrix $\mathbf{\Phi}$ of latent factors. Violations of local independence caused by ratings of the same items and the same raters can be specified either as additional factors appearing in the covariance matrix $\mathbf{\Phi}$ or as a patterned residual covariance matrix $\mathbf{\Psi}$. As argued above, the CSM provides consistent estimates of the mean and covariance structure for ordinal items with misspecified normal distribution likelihood (Arminger & Schoenberg, 1989). This also holds true if the statistical models of G-theory (Brennan, 2001a) are applied to ordinal items because these models are particular cases of CSMs.

2.5 Hierarchical rater model

The GFRM and the CSM model the dependency caused by rating the same items by including an additional random effect or correlated residuals. Hierarchical rater models (HRM; Patz et al., 2002; DeCarlo et al., 2011) assume the existence of a discrete true rating η_{pi} of a person p on an item i. However, the true rating is not observed; rather, it is only indirectly measured by the ratings of several raters. The true rating categories of all items serve as indicators of the person ability θ_p. As a consequence, the item response ratings X_{pir} are hierarchically modeled, given true items η_{pi}, which are also hierarchically modeled, given the person ability θ_p. At the first level, a probability distribution $P(X_{pir} = k|\eta_{pi})$ specifies a rater model, while at the second level, the distribution $P(\eta_{pi} = \eta|\theta_p)$ is specified. At the second level, the GPCM can be chosen for modeling true ratings and can be written as

$$P(\eta_{pi} = \eta|\theta_p) \propto \exp\{\eta\alpha_i\theta_p - b_{ik}\} \tag{11}$$

For the rater model at the first level, two different model specifications have been proposed in the literature. Patz et al. (2002) used a discretized normal distribution as the rater model in the originally proposed hierarchical rater model (HRM; see also Casabianca & Wolfe, 2017):

$$P(X_{pir} = k|\eta_{pi}) \propto \exp\Big(-\frac{1}{2\psi_{ir}^2}[k - (\eta_{pi} + \phi_{ir})]^2\Big) \tag{12}$$

The parameter ϕ_{ir} represents a rater severity parameter that models the systematic displacement of the ratings of rater r from the true rating η_{pi}. The variance parameter ψ_{ir} is a measure of the reliability of the rater. Large values for the variance represent a high precision of the rater. The parameters ϕ_{ir} and ψ_{ir} can also be assumed to be invariant across items if a more parsimonious model should be estimated. We want to emphasize that (11) only parameterizes rater severity and rater imprecision. As noted by Patz et al. (2002), the estimation of severities ϕ_{ir} poses computational challenges for small rater-variances ψ_{ir}.

DeCarlo et al. (2011) proposed a hierarchical rater model based on a latent class signal detection model (HRM-SDT) in which the different scale usage of the raters can also be modeled. The item response probabilities in the rater model are specified as a GRM:

$$P(X_{pir} = k|\eta_{pi}) = G(d_{ir}\eta_{pi} - c_{ir,k+1}) - G(d_{ir}\eta_{pi} - c_{irk}) \tag{13}$$

where d_{ir} are item-rater discriminations and c_{irk} are item-rater-category thresholds. Large values for d_{ir} represent highly discriminating raters. Rater severity/leniency or rater centrality/extremity is represented by different values of the thresholds c_{irk}. Ideal raters, who always agree with the true rating category η, have very large discriminations d_{ir} (e.g., larger than 100) and item thresholds are given as $c_{irk} = d_{ir} \times (k - 0.5)$. It is evident that both hierarchical rater models take the dependence caused by rating the same items into account. The HRM-SDT of DeCarlo et al. (2011) appears to be more

flexible in modeling different rater behavior than the HRM of Patz et al. (2002), although it is possibly more difficult to estimate when only a small amount of data is available. However, neither model takes into account the additional dependence structure that occurs when multiple items are rated by one rater. If halo effects exist in applications, the GMFRM or a model with correlated residuals could be used. Alternatively, the hierarchical rater model can be extended to include an additional dependence structure (see also Wang et al., 2014) or random person-rater effects.

3 Estimation methods and their implementation in R packages

In this section, we present a brief overview of estimation methods that can be used for the rater models introduced in Section 2. We focus on the implementation of these methods in a number of recently released R packages (R Core Team, 2018) written by the authors (**immer**, Robitzsch & Steinfeld, 2018; **TAM**, Robitzsch, Kiefer, & Wu, 2018; **sirt**, Robitzsch, 2018b; **LAM**, Robitzsch, 2018a). This focus is intended to provide a basis for the illustrative examples discussed later; it does not imply general recommendations for real data analyses (see Rusch, Mair, & Hatzinger, 2013, for a more comprehensive overview of R packages for IRT). In general, two broad classes can be distinguished: maximum likelihood (ML) and Bayesian estimation. Several variants of ML estimation are discussed (see also Holland, 1990).

Marginal maximum likelihood (MML) estimation (also labeled as full information maximum likelihood estimation, FIML) estimates model parameters under a distributional assumption about person ability (and further random effects). In most cases, the normal distribution is chosen for person ability. As person ability is a latent variable, it is integrated out in the likelihood that the estimation problem can essentially be reduced to estimating item parameters (and rater parameters) and person distribution parameters (means, variances, and covariances). Essentially, MML operates under the assumption of random persons. Therefore, a person distribution is described by a statistical model and each person is not treated as a fixed entity for which the item response model holds. The expectation maximization (EM) algorithm is often employed for MML estimation (Aitkin, 2016). MML estimation for the Rasch-MFRM is available in the function `TAM::tam.mml.mfr()` of the **TAM** package. Several submodels of the MFRM that allow for different item discriminations can be estimated with `TAM::tam.mml.2pl()` or `sirt::rm.facets()`. An MML implementation of the HRM-SDT model of DeCarlo et al. (2011) can be found in `sirt::rm.sdt()`. In principle, the HRM of Patz et al. (2002) can also be estimated with the MML method, although an implementation is available in any of the R packages discussed in this section. MML estimation for CSMs based on a multivariate normal distribution can be found in the **lavaan** package (Rosseel, 2012) or in the `LAM::mlnormal()` function. G-theory models have equal linear discrimination parameters and fall into the class of linear mixed effects models that can be estimated with the **lme4** package (Bates, Mächler, Bolker, & Walker, 2015).

In joint maximum likelihood (JML) estimation (Lord, 1980; also labeled as fixed effects estimation), person parameters and item parameters are estimated simultaneously. Essentially, persons are treated as fixed and a single parameter is estimated for each person. Hence, no distributional assumption of person ability is needed. The JML estimation is only computationally stable for Rasch-MFRMs and is implemented in the Facets software (Linacre, 1989, 2017). JML has the disadvantage that the number of estimated parameters increases with the number of persons in the sample, which induces the well-known bias in JML estimation (Andersen, 1980). For the PCM, a simple bias-correction formula has been proposed (Andersen, 1980). However, this formula cannot be easily generalized to rating data with complex rating designs in which the number of ratings per person and per item differs. Considering the critique of JML in most of the psychometric literature, resampling methods and analytical methods (Hahn & Newey, 2004) have been proposed, which practically remove the bias caused by JML estimation. Bertoli-Barsotti, Lando, and Punzo (2014) proposed a modification to the likelihood function of the Rasch model for JML estimation that removes most of the bias in item parameters. The reason for the JML bias is that there is no simple way to handle persons with extreme scores (persons score in the lowest category or in the largest category for all items). The so-called ϵ-adjustment method of Bertoli-Barsotti et al. (2014) essentially applies a linear function to the sum score $S_p = \Sigma_i X_{pi}$ in order to map the interval $[0, M_p]$ (M_p is the maximum score for person p) onto $[\epsilon, M_p - \epsilon]$. It should be emphasized that all scores are linearly transformed. The ϵ-adjustment approach is implemented in the `immer::immer_jml()` function of the **immer** package (Robitzsch & Steinfeld, 2018) and extends the method of Bertoli-Barsotti et al. (2014) to polytomous item responses and multiple-matrix designs with arbitrary missing patterns. Therefore, this JML estimation method with bias-correction enables the estimation of the Rasch-MFRM. The statistical properties of the parameter estimates can be seen as being superior to alternative JML implementations of the Rasch-MFRM (for example, in the Facets software; Linacre, 2017). Depending on the application, JML can be substantially faster than MML estimation and, hence, JML could be seen as a viable estimation alternative even if persons are treated as random.

Conditional maximum likelihood (CML; Andersen, 1980) estimation also avoids a specification of the distribution of person ability as person parameters are completely removed in the estimation approach. Hence, CML can be used under the perspective of random persons as well as fixed persons. CML can only be applied for Rasch-MFRMs. The basic idea of CML is that the likelihood of a particular item response pattern with sum score v is conditioned on the sum of the likelihoods of all response patterns with sum score v. It can be shown that the corresponding ratio is independent of person ability and that CML provides consistent item parameter estimates (like MML estimates; van der Linden, 1994). It should be emphasized that CML becomes cumbersome in rating designs in which not all persons are rated by the same items and the same raters because the CML computations must be separately evaluated for every missing data pattern. CML for Rasch-MFRMs is available in the **eRm** package (Mair & Hatzinger,

2007) and the `immer::immer_cml()` function.

MML and CML estimation can be computationally demanding with complex rating data designs because there can be a large number of virtual items with many missing values. To reduce the computational burden, so-called limited information estimation approaches have been proposed, which do not rely on modeling the full item response patterns but, rather, operate on the aggregated information of the data.

The diagonally weighted least squares estimation method (DWLSMV; Muthén, 1984) can be applied to estimate confirmatory factor models for ordinal item responses (e.g., the GRM or GMFRM with a latent variable representation and a probit link function). In this three-stage approach, only the univariate or bivariate frequencies of items (or virtual items, respectively) are used to estimate item thresholds and the polychoric correlations of all items in the first two stages. In the third stage, the item thresholds and the polychoric correlation matrix are estimated as a function of an unknown parameter describing the threshold and covariance structure. DWLSMV estimation can be implemented in the **lavaan** package. In complex rating designs with many raters, not many data are available on virtual items (the response of a particular rater to a particular item) and the estimation of thresholds and polychoric correlations becomes unstable. Therefore, the DWLSMV cannot be reliably applied in these situations.

Composite maximum likelihood estimation (see Varin, Reid, & Firth, 2011, for a review) uses a modified optimization function in such a way that only parts of the data are modeled. We will focus only on the case that specifies a likelihood function for all pairs of items (or virtual items). In contrast to DWLSMV estimation, composite methods are one-stage methods and are applicable to complex rating designs. Composite marginal maximum likelihood estimation (CMML; also labeled as pairwise likelihood estimation) is an estimation method of the confirmatory factor model for ordinal data with a latent variable representation under the probit link function (Katsikatsou, Moustaki, Yang-Wallentin, & Jöreskog, 2012). The estimation is based on the frequencies of the bivariate cross tables of all item pairs. These frequencies are modeled as functions of the model-implied likelihood function, which can be simply evaluated as a function of the unknown model parameters because it can be computed based on the bivariate normal distribution function. Therefore, the estimation method is computationally efficient and arbitrary missing patterns in rating designs can be easily handled. Many variants of the GMFRM in the GRM formulation can be efficiently estimated. Item discriminations, factor covariances, or residual correlations can be estimated as functions of the basis parameters, like in the LLTM, which makes it possible to test the specific hypotheses of rater effects. The CMML estimation approach is implemented in the **lavaan** package and in the `immer::immer_cmml()` function, with a particular emphasis on LLTM representations of the model parameters. The related approach of Garner and Engelhard (2009) is also based on eliminating person parameters by considering pairwise conditional probabilities. However, they propose that model parameters should be estimated by a noniterative algorithm based on eigenvalues on the incidence matrix of pairwise frequencies (the so-called eigenvector method).

As an alternative to CMML, a composite estimation method based on the CML principle can be employed. Composite conditional maximum likelihood estimation (CCML) evaluates the conditional likelihood for pairs of items. Hence, it is also based on only the bivariate information of the dataset. The CCML approach has been proposed for the LLTM for dichotomous data (Zwinderman, 1995) but it can be generalized to polytomous items; our implementation can be found in the `immer::immer_ccml()` function. To this end, Rasch-MFRMs can be estimated more efficiently with CCML than with CML in complex rating designs.

In recent years, Bayesian estimation approaches such as Markov chain Monte Carlo (MCMC) have become very popular due to the availability of very flexible general purpose Bayesian software programs such as BUGS, JAGS, or Stan. In a nutshell, the MCMC approach is a simulation-based stochastic estimation algorithm, which uses random draws of latent variables (person ability, random effects) and model parameters conditional on the information contained in the dataset. The MCMC approach is often seen as being computationally superior to ML estimation for IRT models with many latent variables (Patz & Junker, 1999). In the GMFRM, the random effect person ability as well as the person-item and person-rater effects are estimated. It is relatively easy to estimate this model in a Gibbs sampling approach (Wang et al., 2014). The **immer** package provides a wrapper function for the JAGS software (Plummer, 2003) in the `immer::immer_gmfrm()` function. The HRM of Patz et al. (2002) is mostly estimated with MCMC methods although ML estimation is also possible (DeCarlo et al., 2011). A Metropolis-Hastings within Gibbs sampling algorithm is employed in the `immer::immer_hrm()` function.

It should be emphasized that MCMC estimates are asymptotically equivalent to ML estimates. Hence, MCMC can also be used in applications without a primary focus on Bayesian statistical inference. In IRT models for raters, informative prior distributions decode prior knowledge about parameters in the Bayesian approach. Rater models are often highly parameterized and researchers aim to avoid statistical overfitting. For example, many item-specific rater effects are estimated in a rater model but only practically relevant effects should be signaled by the model. An informative prior normal distribution with a mean of zero and a variance of 0.01 assumes that most rater effects are small. Only those rater effects with large values are estimated as being significantly different from zero (see Muthén & Asparouhov, 2012, for the application of prior distributions in differential item functioning). In an alternative interpretation, model parameters are regularized in such a way that all nonsignificant effects are reduced to zero, which provides a more focused view on the most relevant effects. Similarly, so-called penalized ML estimation has been proposed as a regularization procedure under the ML paradigm for assessing differential item functioning (Tutz & Schauberger, 2015). In the same manner, rater effects can be regularized in a penalized ML approach of a Rasch-MFRM, which will probably be implemented in the TAM package in the near future.

4 Choosing an appropriate rater model

The question of how to choose a suitable model involves an examination of the assumptions, expectations, and properties of the statistical models. In the following, we try to provide a balanced view of advantages and disadvantages of the rater models presented in Section 2.

It has been argued in Section 2 that typical rating designs imply the existence of local dependence caused by person-item and person-rater interactions. While the GMFRM deals with both sources of dependence, the HRM (either in the Patz et al., 2002 or the HRM-SDT specification) only considers additional dependence caused by person-item interactions. It can be argued that, for analytic ratings, halo effects (person-rater interactions) play only a minor role and that therefore, the HRM often fits empirical datasets sufficiently well. The GMFRM and HRM have the advantage that they typically provide a good model (or are at least superior to the MFRM) and provide adequate reliability estimates of person parameters, as sources of local dependence are explicitly modeled. By applying one of the two model classes, a researcher puts substantial emphasis on local dependence because the meaning of all of the model parameters (item parameters, rater effects, and distribution parameters) is coupled with the modeled dependence. In particular, the item and rater parameters in the GMFRM must be interpreted as being conditional on person ability and random person-item and person-rater effects. If the variances of the random item effects substantially differ from each other, item difficulties can no longer be directly compared to each other because they operate on different metrics. A comparison can be made if the random effects are integrated out to form the conditional item response probabilities (see Section 2.3). In addition, the parallel appearance of person ability and random item and rater effects in the GMFRM implies that there is no unique (weighted) maximum likelihood estimator (WLE, Warm, 1989) for the person parameter. Only the mean of the marginal posterior distribution (i.e., the expected value of the posterior distribution, EAP) can be used as a person ability estimate. It should be noted that even in the case of equal discrimination parameters in the GMFRM with random effects, the sum score is no longer a sufficient statistic of the EAP because the ratings are weighted in such a way that ratings corresponding to random effects with smaller variances receive larger weights, while random effects with larger variances receive lower weights. Such a weighting scheme is not always favored, especially in applications in which the person ability estimate is of vital importance for the person itself (e.g., in feedback or in an examination). The HRM and GMFRM are more computationally demanding than unidimensional rater models and this could be seen as a disadvantage for practitioners. We think that this problem can be solved with sufficient computational resources and is not a real limitation in the application of more complex models.

Admittedly, the HRM and GMFRM can probably also not model aspects of the data in order to describe the complex rating behavior. For example, raters can function differently between persons (e.g., Eckes, 2005) or there could be rater drift during a

rating administration (e.g., Leckie & Baird, 2011). Persons are also often clustered within organizational units (e.g., in universities, classes, courses, groups of peers, etc.). This clustering induces additional dependence, which remains nonmodeled in the HRM or GMFRM. However, these aspects are mostly not of major interest in statistical analysis and will be considered as a nuisance (and therefore ignored in the statistical model). Hence, a misspecified likelihood will almost always be the consequence, and pseudo-likelihood estimation is essentially employed, which requires robust ML standard errors (White, 1982). Model parameters resulting from pseudo-likelihood estimation can be interpreted as estimates of some of the population parameters of an assumed statistical model obtained by repeated sampling processes (of persons, raters, clusters, etc.) with comparable assumptions.

In the Rasch-MFRM, the model parameters can be interpreted as being conditional on person ability. The Rasch-MFRM models rater behavior by using a restricted PCM. It has the advantage of computational simplicity as (bias-corrected) JML estimation is computationally fast. Moreover, the sum score of the item responses of a person is a sufficient statistic for the person parameter (WLE, MLE), which facilitates interpretation because of the equal weighting of all the ratings. Rasch-MFRMs assume local stochastic independence and therefore ignore possible dependencies caused by rating the same items or ratings by the same raters. Interestingly, the assumption of local independence in the application of a unidimensional item response model can essentially be reduced to the assumption that residuals cancel out on average. This means that it is assumed that positive and negative local dependence cancel each other out. This assumption is defensible if person ability is interpreted as a major dimension that is statistically extracted from the dataset. Possible violations caused by local independence are regarded as a nuisance factor in statistical modeling. If the sole argument for applying the HRM or the GMFRM is to obtain correct standard errors or adequate reliability estimates for person parameters, we think that this choice is unfounded and that the Rasch-MFRM should be considered instead. The application of the Rasch-MFRM under the local independence assumption should be contrasted with the GMFRM, in which the appearance of random effects only allows for positive local dependence. The nonmodeled positive dependence in the Rasch-MFRM implies that the reliability of the person parameters is underestimated and, therefore, procedures correcting for local dependence have been proposed (Bock, Brennan, & Muraki, 2002). With respect to model parameters such as item difficulties or rater severity effects, robust ML standard errors should be used because the Rasch-MFRM will typically employ a misspecified likelihood function. Notably, this pseudo-likelihood estimation nevertheless provides consistent parameter estimates under repeated sampling assumptions because, asymptotically, the (Kullback-Leibler) distance between a true complex (and unknown) distribution and an assumed parameterized distribution is minimized (White, 1982). As a consequence, the parameter estimates of the Rasch-MFRM for different samples are only comparable (or can only be linked to each other) if similar rating designs are employed that ensure that the extent of (ignored) local dependence remains similar in different samples. When this condition is fulfilled, the use

of the Rasch-MFRM can be justified in applications if the calculation of standard errors for model parameters and person parameters is modified appropriately. This is the case for numerous simulation studies that have aimed to show that applying the Rasch-MFRM to data generated by a GMFRM provides biased parameter estimates (e.g., Wang et al., 2014) because the two models parameterize item response functions in different ways and therefore preclude any legitimate comparison (see Luecht & Ackerman, 2018, for a general discussion about the generalizability of findings from simulation studies in IRT).

It can be expected that a GMFRM including item or rater discrimination parameters will almost always provide a better fit than the Rasch-MFRM. However, we believe that items and raters should be equally weighted as in the Rasch-MFRM because, in applications, the latent construct of interest is defined by having equal contributions of items and persons (see Reckase, 2017 for such a domain sampling perspective). Otherwise, the psychometric model would reweigh the contributions of items and persons in a completely data-driven way, which could be regarded as a threat to validity (Brennan, 2001b). It is sometimes argued in the literature (e.g., Bond & Fox, 2001) that Rasch models have many desirable statistical properties that are not fulfilled in a GMFRM with discrimination parameters (2PL). Maybe a reason for the existence of several myths about the Rasch model could be the property of so-called specific objectivity (Fischer, 1995), which is only guaranteed by the Rasch model and enables the separation of person and item properties in an additive way. Some researchers incorrectly interpret this property as a sample independence of person and item parameters. However, if a statistical model (Rasch or 2PL) holds under the assumption of invariant item parameters (e.g., the same parameters can be applied for specified subpopulations of persons), unbiased comparisons for arbitrary selections of items are possible for both Rasch and 2PL models. The Rasch model has the distinctive advantage that, due to the existence of the sufficient statistic of the sum score, CML estimation can be conducted. However, CML estimation and MML estimation, usually performed for 2PL models, will both provide consistent parameter estimates. Therefore, some researchers' preference for the Rasch model instead of the 2PL model can statistically only be justified by the feasibility of CML estimation (see van der Linden, 1994, for more detailed arguments), but CML is inferior to MML estimation in finite samples. In summary, we believe that the advantage of using the Rasch-MFRM can only be argued by using validity reasons related to the equal weighting property and to the ease of parameter interpretation; we do not believe that it can be argued that the Rasch-MFRM has superior statistical and measurement-related properties.

The rater models discussed above place person ability and model parameters onto a metric of a latent variable, namely, the logit metric or a probit metric. Sometimes, it is preferable to use the original metric of raw scores for interpretational purposes. This seems to be particularly true for research settings in which people with less training in psychometrics are involved. As it has been argued in Section 2.4, the application of CSMs or G-theory models to ordinal data structurally represents the mean and covariance structure of the data and provides consistent parameter estimates, although the assumed normal distribution is misspecified. An important application is the computation of fair

scores (see Eckes, 2015), which adjust person parameter estimates for systematic rater effects. While the use of fair scores in the logit metric of the Rasch-MFRM entails a bias at the boundary of ratings scales (especially for datasets with only few ratings per person), employing the original metric by using a normal distribution model avoids this bias.

Finally, the role of the fit of particular entities (items, raters, persons) or of the whole model has to be considered. From a strictly psychometric perspective, the application of the model fit of an IRT model from a random persons perspective treats model fit as the discrepancy between an observed and a model-implied covariance structure with respect to the items (or virtual items). Therefore, items are considered as being fixed and nonexchangeable, and a possible replication of the experiment must involve the same items and same raters (Brennan, 2011). The application of the G-theory (or classical test theory, CTT) only makes assumptions about random sampling with respect to persons, items, and raters. As the samples are thought to be representative with respect to corresponding populations, all observations have to be equally weighted in the statistical model. It seems that the adequacy of applying G-theory models with equal discrimination parameters can be tested against the application of models in which different discrimination parameters are allowed. However, the perspective of fit does not play a role in the G-theory as the model is only intended to represent the sampling process. Hence, G-theory models or CTT models essentially require fewer assumptions than IRT models (see Brennan, 2011) and, therefore, they allow for broader generalizations. Unfortunately, this fact is often overlooked in applied research and even in parts of the psychometric literature.

To sum up, we have discussed the possible arguments for choosing one of the classes of models for human ratings. These models have different assumptions, which can often be simultaneously defended for a single dataset under different research perspectives or with different uses of model parameters. Applied researchers should be cautious of the psychometric literature that promotes the superiority of one model class over another and justifies its recommendations mainly based on the results of simulation studies.

5 Empirical application in R

In this section, we illustrate the application of several IRT models on a sample dataset and show how they can be estimated within R. The sample dataset is contained in the **immer** package and has the name `data.ptam4`. It comprises 592 ratings for a single essay written by 209 students and rated by ten raters on three items. 39 students were rated by all ten raters, one student by nine raters, 17 students received ratings from two, three or four raters, and 152 students had only ratings from a single rater. Each row in the sample dataset `data.ptam4` includes all ratings of a rater on all items corresponding to an essay of a student. The structure of the dataset can be inspected in R by using the `head()` function.

It can be seen that the student with identifier (variable `idstud`) 10010 has two rows in the dataset which means that she or he received ratings from two raters (variable `rater`) 844 and 802. Ratings were provided on three items `crit2`, `crit3` and `crit4` on a four-point scale (with integer values 0, 1, 2, and 3).

Complete syntax for the specification of all models in this section is provided by a vignette which is included in the **immer** package.

```
R> data(data.ptam4, package="immer")
R> dat <- data.ptam4
R> head(dat)
```

```
  idstud rater crit2 crit3 crit4
1  10005   802     3     3     2
2  10009   802     2     2     1
3  10010   844     0     1     2
4  10010   802     2     2     1
5  10014   837     1     2     2
6  10014   824     0     2     2
```

Item response models for a single item

Before analyzing the complete rating dataset with three items, we investigate rater effects based on only a single item "crit2". We use a dataset in a so called wide format in which columns refer to ratings of a single rater. In our analysis, we use ratings of 40 students who received multiple ratings from ten raters. Only one student was unintentionally rated by only nine raters. The dataset can be attached as `data.ptam4wide` from the **immer** package.

Table 1:
Descriptive Statistics for Item "crit2"

Rater	Cat0	Cat1	Cat2	Cat3	M	SD	Cor
R802	.10	.38	.38	.15	1.58	0.87	.76
R803	.33	.38	.18	.13	1.10	1.01	.79
R810	.20	.38	.33	.10	1.33	0.92	.86
R816	.25	.25	.35	.15	1.40	1.03	.80
R820	.03	.46	.38	.13	1.62	0.75	.69
R824	.23	.40	.25	.13	1.28	0.96	.78
R831	.18	.33	.35	.15	1.48	0.96	.87
R835	.38	.28	.23	.13	1.10	1.06	.76
R837	.08	.30	.35	.28	1.83	0.93	.79
R844	.30	.25	.25	.20	1.35	1.12	.78

Note: Cat0, Cat1, Cat2, Cat3 = relative frequencies for categories 0, 1, 2, 3; Cor = correlation of rating of a single rater with average score across all raters

Table 1 displays descriptive statistics for the rating dataset for item "crit2". Category frequencies, the mean, the standard deviation and the correlation of a rating with the aver-

age score across all ratings are shown in the table. By comparing the means, it is evident that Raters 803 and 835 are severe while Rater 837 is lenient. Moreover, by inspecting relative frequencies and the standard deviations, Rater 820 shows a centrality tendency while Rater 844 can be characterized by an extremity tendency. Finally, by considering the correlation with the average score, Rater 820 exhibits the lowest agreement, while Raters 810 and 831 show the largest extent of agreement.

In the next step, we apply several item response models to the rating dataset involving the single item "crit2" (see Wolfe, 2014, M. Wu, 2017 for applications of this approach). In this approach, an item refers to a single rater and each rater is parameterized by its own set of parameters. First, it is assumed that a continuous variable is used for modelling the ratings of the four-point scale item. We fit the PCM with an assumption of homogeneous rater effects (i.e., all raters possess the same set of parameters), the PCM, and the GPCM (Models M01, M02, and M03). Second, we follow the principle of the HRM in which true ratings of an item are modelled. Therefore, we specify located latent class Rasch models (LOCLCA; Formann, 1985) which parameterize the response functions of the raters by the PCM but assume a discrete ability variable. The locations of these latent classes on the θ metric and the class probabilities are estimated. In our analysis, we fit LOCLCA with three, four and five latent classes (Models M13, M14, and M15). For the four-point scale item, a LOCLCA with four latent classes would be expected if true ratings can be empirically identified.

Table 2:
Model Comparisons for Item Response Models for Item "crit2"

Label	Model	Deviance	#par	AIC	BIC
M01	PCM equal	861.03	4	869	**876**
M02	PCM	**785.58**	31	**848**	900
M03	GPCM	774.45	40	854	922
M13	LOCLCA(3)	808.14	34	876	934
M14	LOCLCA(4)	775.52	36	848	**908**
M15	LOCLCA(5)	**769.24**	38	**845**	909

Note: #par = number of estimated parameters; PCM equal = partial credit model in which parameters for all raters were constrained to be equal; LOCLCA(k) = Located class analysis with k located latent classes and the PCM is used as the item response function.

Table 2 contains deviances and information criteria for the fitted models. Model selection can be conducted by using differences of deviance values of nested models and performing a likelihood ratio test (LRT) or by considering models with smallest information criteria AIC and BIC. When comparing models M01, M02 and M03, it turned out that the model with equal parameters for raters must be rejected which means that raters differ with respect to their rating behavior. The GPCM did not fit the data significantly better than the PCM although the small sample size ($N = 40$) has to be considered. As an example, we show how to fit the PCM using the **TAM** package and discuss parts of the summary

output (dat2 is the dataset data.ptam4wide).

```
R> items <- c("crit2","crit3","crit4")
R> mod02 <- TAM::tam.mml(  resp=dat2[,items] , irtmodel="PCM2")
R> summary(mod)
```

```
Item Parameters -A*Xsi
   item  N     M xsi.item AXsi_.Cat1 AXsi_.Cat2 AXsi_.Cat3 B.Cat1.Dim1
1  R802 40 1.425    0.274     -2.923     -2.414      0.823           1
2  R803 40 1.300    0.714     -1.033     -0.730      2.142           1
3  R810 40 1.500   -0.086     -2.960     -2.047     -0.257           1
4  R816 40 1.600   -0.401     -3.892     -3.814     -1.203           1
5  R820 39 1.513   -0.011     -4.095     -3.747     -0.033           1
6  R824 40 1.450    0.333     -2.867     -2.745      0.998           1
7  R831 40 1.425    0.180     -2.263     -1.177      0.540           1
8  R835 40 1.450    0.228     -3.922     -3.602      0.683           1
9  R837 40 2.075   -2.073     -5.491     -7.872     -6.219           1
10 R844 40 1.600   -0.199     -2.812     -2.944     -0.597           1
```

The argument irtmodel="PCM2" requests the Andrich (1978) parameterization of the PCM. The column xsi.item contains the item difficulty of the PCM which can be interpreted as rater severity/leniency. The most lenient Rater 837 has the smallest item difficulty (i.e., rater difficulty) while for the most severe Rater 803 the largest parameter was obtained. The columns AXsi_.Cat1, AXsi_.Cat2 and AXsi_.Cat3 include rater-category parameters which assess aspects of severity/leniency or centrality/extremity behavior of the raters. The rater parameters can most easily be interpreted by computing Thurstonian thresholds from the PCM using the TAM::tam.threshold() function.

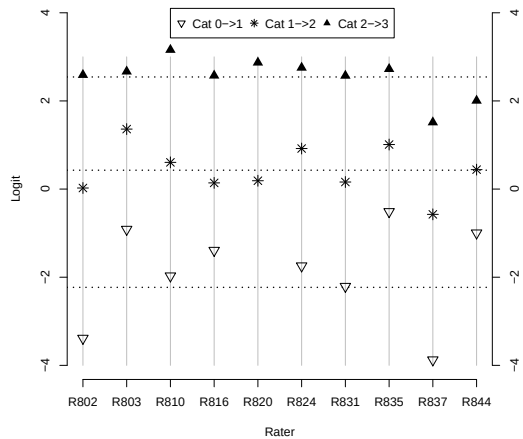


Figure 2:
Rater thresholds from the PCM (Model M02)

In Figure 2, the thresholds for all raters and all categories are depicted. It is evident that

Raters 802 and 837 are lenient with respect to using the zero category while the opposite is true for Rater 835. Summing up, it can be seen that the variability of the thresholds among raters between 0 and 1 is much larger than for the thresholds between 1 and 2 and 2 and 3. This means that the raters show less agreement for rating students in lower categories, but more agreement in rating higher categories.

We also compute infit statistics for raters (Eckes, 2015) based on the PCM (Model M02) using the `TAM::msq.itemfit()` function. Raters 810 and 831 which showed the highest agreement with the average rating (see Table 1) have the lowest infit statistics (.77 and .75, respectively) which can be interpreted as overfit. The largest infit statistics were observed for Raters 835 and 844 (1.14 and 1.19, respectively) which indicates underfit of these two raters. The GPCM can be fitted using the `TAM::tam.mml.2pl()` function using the argument `irtmodel="GPCM"`. It turned out that Raters 810 and 831 have the largest rater discriminations (4.25 and 4.39, respectively).

The model fit for different LOCLCAs are shown in Table 2. It should be emphasized that the LOCLCA with four or five latent classes has a slightly superior fit to the PCM which assumes a continuous ability. Although the LOCLCA with four classes could be preferred because it can be more easily interpreted, we present the results of the LOCLCA with five classes. The LOCLCA can be fitted using the `TAM::tamaan()` function which allows the specification of IRT models similarly to the **lavaan** package.

```
R> tammodel <- "
R+ ANALYSIS:
R+ TYPE=LOCLCA;   # type of the model
R+ NCLASSES(5);   # 5 classes
R+ NSTARTS(10,30); # 10 random starts with 30 iterations
R+ LAVAAN MODEL:
R+   F =~ R802__R844
R+     "
R> mod15 <- TAM::tamaan( tammodel , resp=dat2 )
R> summary(mod)
```

```
Cluster locations
        V1  prob
Cl1 -9.990 0.048
Cl2 -2.814 0.108
Cl3 -0.124 0.334
Cl4  1.463 0.335
Cl5  3.415 0.175
------------------------------------------------------------
Item Response Probabilities
   item itemno Cat Class1 Class2 Class3 Class4 Class5
1  R802      1   0 0.9988 0.3835 0.0284 0.0024 0.0000
2  R802      1   1 0.0012 0.5975 0.6526 0.2641 0.0242
3  R802      1   2 0.0000 0.0190 0.3056 0.6048 0.3903
4  R802      1   3 0.0000 0.0001 0.0133 0.1288 0.5855
```

```
[...]
------------------------------------------------------------
Class-Specific Item Means
   item itemno Class1 Class2 Class3 Class4 Class5
1  R802      1 0.0012 0.6357 1.3038 1.8600 2.5612
2  R803      2 0.0001 0.0747 0.6278 1.3383 2.4807
3  R810      3 0.0002 0.2063 1.0124 1.6340 2.3854
4  R816      4 0.0001 0.1067 0.9765 1.8310 2.5666
5  R820      5 0.0553 1.0133 1.2978 1.7869 2.4802
6  R824      6 0.0001 0.1668 0.9042 1.5541 2.4835
7  R831      7 0.0002 0.2583 1.1573 1.8270 2.5646
8  R835      8 0.0000 0.0455 0.5335 1.4349 2.4938
9  R837      9 0.0027 0.8269 1.5150 2.2048 2.8086
10 R844     10 0.0001 0.0719 0.7754 1.8169 2.7131
```

The latent classes from the model output can be interpreted as latent ratings. By inspecting item response probabilities and class-specific item averages, latent classes 1 and 2 can be associated with "true" category 0, and latent classes 3, 4 and 5 can be associated with "true" categories 1, 2 and 3. Note that Class 1 includes students which were (very probably) rated as 0 by all raters while raters differed in their ratings for students in Class 2. Raters 802, 820 and 837 rated a substantial portion of students in Class 2 into categories 1, 2 or 3 while all other raters mostly rated students into category 0. Moreover, from the output it can be also concluded that Rater 837 is the most lenient one.

G-theory models

In the following analyses, we use the full datasets including three items, ten raters and 209 students. As a preliminary analysis to more complex item response models, we fit G-theory models (specified as linear mixed effects models) for assessing the amount of variance which can be attributed to different sources. We estimate G-theory models using the **lme4** package. In order to achieve this, the dataset has to be converted into a long format in which one row refers to the combination of a student, a rater and an item. The needed structure has already been prepared as the dataset `data.ptam4long` in the **immer** package. Four different G-theory models are fitted (Models M21, M22, M23 and M24). The first three models assume homogeneous variance components (for random effects of items or raters) while the last model allows for item-specific or rater-specific variances of random effects. The G-theory Model M23 including person, person-item and person-rater random effects can be estimated using the following syntax (`value` denotes the variables which include all ratings for students, items and raters)

```
R> mod23 <- lme4::lmer( value ~ rater*item + ( 1 | idstud ) +
R+      ( 1 | idstud:item ) + ( 1 | idstud:rater), data = data.ptam4long )
R> summary(mod23)
```

```
Random effects:
 Groups       Name        Variance Std.Dev.
 idstud:item  (Intercept) 0.06497  0.2549
 idstud:rater (Intercept) 0.09344  0.3057
 idstud       (Intercept) 0.28119  0.5303
 Residual                 0.21512  0.4638
Number of obs: 1776, groups: idstud:item, 627; idstud:rater, 592; idstud, 209
```

In this model, rater-specific item means are allowed (fixed effects `item*rater`). It can be seen from the output that a large part of the variance corresponds to student ability. Interestingly, the variance component due to person-rater interactions (i.e., halo effects) is slightly larger than the amount of dependence due to person-item interactions. This finding sheds some light on the application of HRM which only handles dependency due to random item effects but not to random rater effects.

Table 3:
Variance Component Estimates from G-theory Models

Variance	Model M21	Model M22	Model M23
p	.331	.323	.281
$p \times i$	—	.044	.065
$p \times r$	—	—	.093
Residual	.334	.299	.215

Note: p = persons; i = items; r = raters

In Table 3, the variance component estimates for the first three models are shown. When comparing Model M21 and M22, it can be seen that most part of the variance of the item effect ($p \times i$) is confounded with the residual variance in Model M21. When including the random rater effect ($p \times r$) in Model 23, a substantial part of the true score variance is captured which shows that neglecting dependency due to halo effects results in overly optimistic reliability estimates because the true score variance is overestimated.

Finally, we show how to estimate a G-theory model with heterogeneous variance components (Model M24). The specification is a bit cumbersome when done manually because dummy variables for all items (e.g., `I_crit2`) and all raters (e.g., `R_802`) are involved in the model specification.

```
R> mod24 <- lme4::lmer( value ~ rater * item + (1 | idstud) +
R+    (0 + I_crit4 | idstud:item) +  (0 + I_crit3 | idstud:item) +
R+    (0 + I_crit2 | idstud:item) +  (0 + R_844 | idstud:rater) +
R+    (0 + R_837 | idstud:rater) +  (0 + R_835 | idstud:rater) +
R+    (0 + R_831 | idstud:rater) +  (0 + R_824 | idstud:rater) +
R+    (0 + R_820 | idstud:rater) + (0 + R_816 | idstud:rater) +
R+    (0 + R_810 | idstud:rater) +  (0 + R_803 | idstud:rater) +
R+    (0 + R_802 | idstud:rater), data= data.ptam4long)
```

The variance component estimates of person-item interactions (`idstud:item`) were estimated as .094 (Item “crit2”), .000 (Item “crit3”) and .092 (Item “crit4”) showing that

no local dependency was introduced for "crit3". The variance component estimates of person-rater interactions (`idstud:rater`) showed a considerable amount of variation (M= .096, SD=.059, Min=.027, Max=.204).

Many-facet rater models

In this subsection, we illustrate the application of several MFRMs. In a first series of models, we fit Rasch-MFRMs which assume equal item and rater discrimination parameters (Models M31, ..., M36). In a second series of models, we fit MFRMs which allow the inclusion of item and rater discrimination parameters (Models M41, ..., M46).

In a Rasch-MFRM, the item response function for person p, item i, rater r and category k is given as $P(X_{pir} = k|\theta_p) \propto \exp(k\theta_p - b_{irk})$. Different constrained versions for parameters b_{irk} can be estimated. These versions can be defined using design matrices or – more conveniently – using the formula language in R when fitting Rasch-MFRMs with the `TAM::tam.mml.mfr()` function in the **TAM** package. For example, the formula `~ item*step + rater` for facets items, raters and steps (i.e., categories) corresponds to the constraint $b_{irk} = b_{ik} + b_r$. In principle, formulas of arbitrary complexity and an arbitrary number of facets can be specified in the **TAM** package using the argument `formulaA`. The Rasch-MFRM `~ item*step + rater` (Model M32) can be estimated using the following syntax (`dat` is the dataset `data.ptam4`)

```
R> facets <- dat[, "rater", drop=FALSE ]
R> mod32 <- TAM::tam.mml.mfr( dat[,items], facets=facets,
R+             formulaA = ~ item*step + rater, pid=dat$pid )
R> summary(mod32)
```

```
Item Facet Parameters Xsi
[...]
7     rater802     rater -0.118  0.101
8     rater803     rater  1.247  0.101
9     rater810     rater -0.052  0.099
10    rater816     rater -0.017  0.101
11    rater820     rater -0.412  0.099
12    rater824     rater  0.024  0.099
13    rater831     rater  0.169  0.100
14    rater835     rater  0.666  0.100
15    rater837     rater -1.483  0.099
16    rater844     rater -0.023  0.300
[...]
```

The function automatically creates virtual items for estimating the constrained PCM. The estimated item and rater parameters can be found in output section `Item Facet Parameters Xsi` (only rater parameters are displayed). The main rater effects in this section indicate the extent of leniency/severity tendencies. These rater effects are almost perfectly correlated with the means for each rater (across all items) which can be expected because these means are sufficient statistics for the rater effects.

Table 4:
Model Comparisons of Different Rasch-MFRMs

Label	formulaA	Deviance	#par	AIC	BIC
M31	~ item*step	3802.42	10	3822	**3866**
M32	~ item*step+rater	3763.23	19	3801	3885
M33	~ item*step+rater*step	3693.46	37	3767	3930
M34	~ item*step+rater*item	3699.89	37	3774	3936
M35	~ item*step+rater*item+rater*step	3632.30	55	**3742**	3983
M36	~ item*rater*step	**3562.38**	91	3744	4143

Note: formulaA = formula specification of Rasch-MFRM; #par = number of estimated parameters.

In Table 4, model comparisons of different Rasch-MFRMs are shown. It can be seen that model fit improves when interaction effects of raters and items or raters and categories are included. The most complex model which assumes a PCM for all virtual items based on combinations of items and raters would be favored based on a LRT but not based on information criteria. It should be noted that information criteria are not to be expected to provide valid statistical inference in case of incomplete designs[1]. This finding highlights that the specification of rater models should not stop with modelling severity/leniency tendencies as other rater effects can be of similar or larger importance.

In a second series of models, we investigate whether differences in discriminations of items or raters can be found. Based on the item response function $P(X_{pir} = k|\theta_p) \propto \exp(ka_{ir}\theta_p - b_{irk})$, different specifications for the discrimination parameter a_{ir} are employed. These models can be estimated using the `sirt::rm.facets()` function. Different specifications for the discrimination parameters can be requested by using the arguments `est.a.item` and `est.a.rater`. The following syntax shows how to estimate Model M44 (see also Table 5) which includes item and rater discriminations in a multiplicative way (i.e., $a_{ir} = a_i a_r$). A model based on virtual items in which all PCM (or GPCM) parameters are estimated can be requested by the argument `rater_item_int=TRUE`.

```
R> mod44 <- sirt::rm.facets( dat[ , items], rater=dat$rater, pid=dat$pid,
R+           est.a.item=TRUE, est.a.rater=TRUE, reference_rater="831",
R+           rater_item_int=FALSE)
R> summary(mod44)

Item Parameters
   item   N     M tau.Cat1 tau.Cat2 tau.Cat3     a  delta delta_cent
1 crit2 592 1.409   -2.244   -2.053    0.542 0.889  0.181      0.368
```

[1] A large fraction of students in the dataset only received a single rating. With three items on a four-point scale, 10 parameters can be estimated for these students (9 item parameters and 1 variance parameter). However, in the Rasch-MFRM specification all students are penalized in the AIC formula by the total number of parameters which refers to a response pattern for students which have received multiple markings from all raters (90 item parameters, 1 variance parameter). Therefore, the number of estimated parameters in the AIC formula must be an overestimate of penalization in incomplete designs.

```
2 crit3 592 1.586   -5.166   -5.702   -1.675 1.475 -0.558     -0.371
3 crit4 592 1.508   -3.342   -3.299   -0.555 0.762 -0.185      0.003
-----------------------------------------------------------------------
Rater Parameters
   rater   N     M      b     a thresh b.cent a.cent
1    802 174 1.540  0.147 0.989  0.146  0.095  1.053
2    803 183 1.158  0.959 1.263  1.211  0.906  1.327
3    810 183 1.508 -0.306 0.972 -0.298 -0.358  1.036
4    816 171 1.503  0.264 0.972  0.257  0.212  1.035
5    820 180 1.606 -0.544 0.862 -0.469 -0.597  0.926
6    824 189 1.492  0.129 1.093  0.141  0.077  1.157
7    831 177 1.446  0.000 1.000  0.000 -0.052  1.064
8    835 171 1.298  0.782 0.605  0.473  0.730  0.669
9    837 180 1.944 -1.049 0.626 -0.656 -1.101  0.690
10   844 168 1.512  0.140 0.980  0.137  0.088  1.044
```

From the output, we see that item "crit3" is more discriminative than the other two items (see column `a`). Further, Raters 803 and 824 are most discriminative (i.e., accurate) while Raters 835 and 837 are least discriminative (i.e., inaccurate) (see column `a.cent`). We also calculated rater infit statistics from the Rasch-MFRM Model M32 (`~ item*step + rater`) and compared these with rater discriminations from Model M42 ($b_{irk} = b_{ik} + b_r$, $a_{ir} = a_r$). Lower rater discriminations tended to result in higher rater infit values ($r = -.33$). It turned out that the relationship of both statistics was stronger ($r = -.59$) when an outlying observation (Rater 803) was removed from the calculation.

Table 5:
Model Comparisons of Different MFRMs

Label	b_{irk}	a_{ir}	Deviance	#par	AIC	BIC
M41	$b_{ik} + b_r$	1	3763.23	18	3799	**3878**
M42	$b_{ik} + b_r$	a_r	3738.55	26	3791	3905
M43	$b_{ik} + b_r$	a_i	3750.96	20	3791	3879
M44	$b_{ik} + b_r$	$a_i a_r$	3724.43	29	3782	3910
M45	b_{irk}	1	3699.89	37	3774	3936
M46	b_{irk}	a_{ir}	**3609.85**	64	**3738**	4018

Note: b_{ir} = specification of item-specific rater intercept; a_{ir} = specification of item-specific rater discrimination #par = number of estimated parameters.

Finally, the model comparison from Table 5 indicated that the most flexible model parameterizing all virtual items by the GPCM would be preferred based on the LRT and AIC. In summary, it can be concluded that Rasch-MFRMs allowing for interaction effects of raters with item or categories or MFRMs with rater discriminations should be preferred from the perspective of model fit to a Rasch-MFRM in which only a main rater severity effect is modelled. We expect that this conclusion will not change if measures of approximate model fit would be employed. This modelling exercise illustrates our argument that a preference of a simpler Rasch-MFRM can (in most applications) only be

justified for validity reasons, that is, when the differential weighting of items and raters by a psychometric model is not warranted.

Generalized many-facet rater models

By employing G-theory models it was observed that there is a substantial amount of variance attributed to person-item and person-rater interactions. Now, a series of GFRMs is fitted in which we allowed item discriminations and we included particular variance components. Four models were specified. Model M51 only contains the random person effect. Model M52 additionally includes the random item effect while in Model M53 the random rater effect is included. Finally, we include all three variance components in Model M54. The **immer** package provides a wrapper function `immer::immer_gmfrm()` to the JAGS software (Plummer, 2003). Model M54 can be estimated using the following syntax.

```
R> mod54 <- immer::immer_gmrfm(dat[,items], rater=dat$rater, pid=dat$idstud,
R+              fe_r="r", re_pi=TRUE, re_pr=TRUE, iter=iter, burnin=burnin)
```

The argument `fe_r` specifies the fixed effects structure of intercepts b_{irk} of the IRT model with respect to raters. Options are `"n"` ($b_{irk} = b_{ik}$), `"r"` ($b_{irk} = b_{ik} + b_r$), `"ir"` ($b_{irk} = b_{ik} + b_{ir}$), `"rk"` ($b_{irk} = b_{ik} + b_{rk}$) or `"a"` (all effects are specified, i.e. all b_{irk} are estimated without constraints). The arguments `re_pi` and `re_pr` indicate whether random effects should be included in the GMFRM. For example, Model M52 can be estimated using `re_pi=TRUE` and `re_pr=FALSE`. We use 50,000 iterations (argument `iter`) and 10,000 burn-in iterations (argument `burnin`) which provided a good convergence behavior of the MCMC estimation approach in our example.

We only briefly discuss the results of Model M54. The variance component estimates varied considerably among items ("crit2": .12, "crit3": .05; "crit4": .66). The rater severities correlated highly with the average rater scores ($r = -.99$) and did also show some variation among raters (SD=.48, Min=-1.06 [Rater 837], Max=0.84 [Rater 803]). The variance estimates for person-rater interactions also exhibited some variability among raters (M=.27, SD=.32). Two Raters 844 (.53) and 803 (1.09) had remarkably high variance estimates indicating that halo effects were strongly present for these raters. Finally, the correlation of the rater variances from the GMFRM (Model M54) and from the G-theory model (Model M24) was .84 indicating that findings remain relatively stable irrespectively of whether the logit or the original metric is chosen.

Hierarchical rater model based on signal detection theory

In the GMFRM, local dependence is taken into account by including additional random effects. In the HRM, ratings are modelled by a hierarchical approach which first assumes that manifest ratings are modelled conditionally on true discrete ratings (signal detection model, SDM). Second, true ratings are modelled by item response functions (item response model, IRM). For both models different specifications can be chosen.

Table 6:
Model Comparisons of Different HRM-SDT Models

Label	IRM	SDM	Deviance	#par	AIC	BIC
M61	PCM	`n`	3540.53	10	3561	3594
M62	PCM	`e`	3530.70	14	3559	3605
M63	PCM	`r`	3314.19	50	3414	3581
M64	PCM	`a`	**3135.89**	130	**3396**	3830
M71	GPCM	`n`	3525.08	12	3549	3589
M72	GPCM	`e`	3512.23	16	3544	3598
M73	GPCM	`r`	3298.47	52	3402	**3576**
M74	GPCM	`a`	3135.18	132	3399	3840

Note: IRM = specified item response model; SDM = specified signal detection model (`n` = no effects; `e` = exchangeable effects for items and raters; `r` = rater effects; `a` = all effects); #par = number of estimated parameters.

In Table 6, different specifications of our fitted models are shown. The IRM uses either the PCM or the GPCM. In the SDM, discrimination parameters d_{ir} and intercept parameters c_{irk} are estimated with several constraints. Regarding our sample dataset it turned out the PCM with a SDM, in which all rater parameters were allowed to be item-specific, showed the best fit (Model M64) in terms of the LRT and AIC.

For facilitating the interpretation, we focus on the discussion of results of Model M63 in which rater effects in the SDM are assumed to be independent of items. The HRM-SDT can be estimated using the `sirt::rm.sdt()` function. To choose the GPCM instead of the PCM one has to use the argument `est.a.item=TRUE`. Different specifications of the SDM can be chosen by using the arguments `est.c.rater` and `est.d.rater`. The estimation of Model M63 can be conducted using the following syntax.

```
R> mod63 <- sirt::rm.sdt( dat[,items], rater=dat$rater, pid=dat$idstud,
R+                est.c.rater="r" , est.d.rater="r")
R> summary(mod63)
```

```
Rater Parameters
   item.rater  N     M     d    c_1    c_2    c_3 c_1.trans c_2.trans c_3.trans
1   crit2-802 58 1.655 4.939  2.590  7.356 13.000     0.524     1.489     2.632
2   crit2-803 61 1.000 3.564  3.264  6.525  9.801     0.916     1.831     2.750
3   crit2-810 61 1.344 4.529  1.213  6.819 11.463     0.268     1.506     2.531
4   crit2-816 57 1.526 5.553  3.336  7.897 14.463     0.601     1.422     2.605
5   crit2-820 60 1.650 7.967 -1.182 11.471 21.195    -0.148     1.440     2.661
6   crit2-824 63 1.365 7.279  4.593 10.668 19.034     0.631     1.466     2.615
7   crit2-831 59 1.373 4.509  1.449  7.464 11.728     0.321     1.655     2.601
8   crit2-835 57 1.035 2.858  1.526  4.762  8.564     0.534     1.666     2.996
9   crit2-837 60 1.833 2.857 -1.314  2.454  6.524    -0.460     0.859     2.283
10  crit2-844 56 1.304 3.679  2.004  4.851  9.323     0.545     1.319     2.534
```

We focus on the interpretation of rater parameters. Raters 820 and 824 are most reliable

because high discrimination parameters d_r were estimated for them. Further, Raters 835 and 837 are least reliable. Severity/leniency and centrality/extremity tendencies can be identified by the intercept parameters c_{rk}. The relative criteria locations $c^*_{rk} = c_{rk}/d_r$ (displayed as `c_1.trans`, `c_2.trans`, `c_3.trans` in the output) indicate the relative "difficulty" for every category of a rater. For raters which do not produce systematic biases, relative criteria locations of .5, 1.5, and 2.5 would be expected for four-point scale items. In Figure 3, these locations are displayed for all raters and all criteria. It can be seen that Raters 820 and 837 are lenient with respect to rating students into the zero category. Rater 803 is more severe because she or he more frequently rates students into the zero category. The standard deviation among raters of relative criteria locations can be computed to assess the uncertainty of rating particular categories. Differentiating students between 0 and 1 showed most variability (SD=.40), while the SD for categories 1 and 2 (SD=.26) and 2 and 3 (SD=.18) was lower. It should be emphasized that a measure of rater severity can be calculated from HRM-SDT output by averaging criteria locations, i.e. computing $\overline{c}^*_r = \sum_k c^*_{rk}/K$.

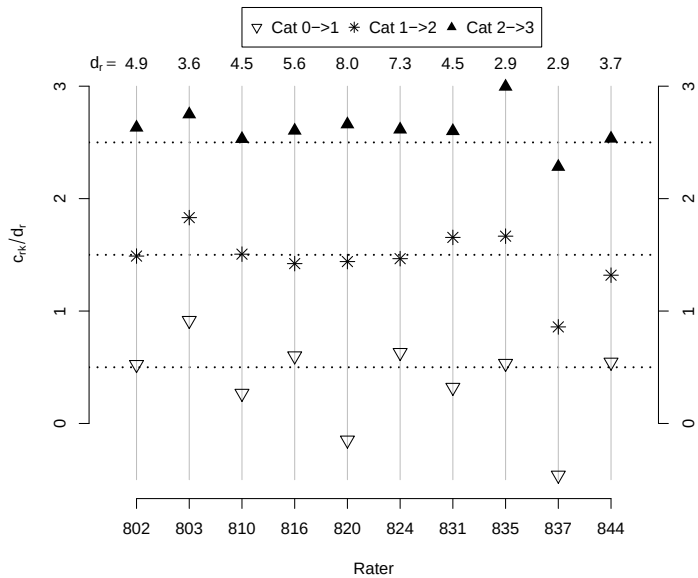


Figure 3: Plots of the relative criteria locations $c^*_{rk} = c_{rk}/d_r$ for the HRM-SDT (Model M63). The solid horizontal lines show intersection points for the underlying distributions.

Hierarchical rater model of Patz et al. (2002)

Finally, we want to fit the alternative HRM of Patz et al. (2002). This model includes rater severity (rater bias) ϕ_{ir} and rater variance ψ_{ir} as rater parameters. Two models are fitted. First, Model M81 assumes that the rater parameters are item independent while in Model

M82 these parameters are specified to vary across items. Different specifications can be chosen by using the arguments `est.phi` and `est.psi` in the `immer::immer_hrm` function. Both models employ the PCM as the IRM. Model M81 can be estimated using the following syntax based on 500,000 iterations and 200,000 burn-in iterations[2].

```
R> mod81 <- immer::immer_hrm( dat[,items], pid=dat$idstud, rater=dat$rater,
R+             est.phi="r", est.psi="r", iter=iter,  burnin=burnin)
R> summary(mod81)

Rater Parameters
    item rater rid N_Rat     M    phi   psi
1  crit2   802   1    58 1.655  0.107 0.383
2  crit2   803   2    61 1.000 -0.303 0.644
3  crit2   810   3    61 1.344  0.156 0.334
4  crit2   816   4    57 1.526  0.078 0.451
5  crit2   820   5    60 1.650  0.208 0.212
6  crit2   824   6    63 1.365  0.035 0.321
7  crit2   831   7    59 1.373  0.057 0.387
8  crit2   835   8    57 1.035 -0.130 0.725
9  crit2   837   9    60 1.833  0.629 0.522
10 crit2   844  10    56 1.304  0.135 0.635
```

The SD of student ability was estimated as 9.42 and was surprisingly high. In the HRM-SDT, a much lower SD of 3.05 was obtained in Model M63. It can be seen in the output of Model M81 that Rater 837 is most lenient because she or he has the highest ϕ value while Rater 803 is most severe. Rater 820 gives the most accurate ratings because she or he has the lowest rater variability ψ while Rater 835 is the least accurate.

The results of the HRM of Patz et al. (2002) (Model M81) should now be compared with the HRM-SDT (Model M63). The correlation of rater precision (i.e., $1/\psi_r$) and rater discrimination (d_r) was relatively high ($r = .88$) indicating that both models reach similar conclusions. Moreover, we correlated the rater severity ϕ_r with the average relative criteria location $\overline{c}_r^*$ of the HRM-SDT. We obtained an almost perfect correlation of $r = -.99$. Therefore, the HRM-SDT also proves useful in assessing rater severity.

Finally, we briefly discuss interesting findings of Model M82. In this HRM, rater severity and rater variances are item-specific. One could question whether all item-rater interaction effects need to be specified. To this end, a F-test based on the MCMC output can be conducted for testing the hypothesis of equal rater severity among items ($\phi_{1r} = \phi_{2r} = \phi_{3r}$) and of equal rater variance ($\psi_{1r} = \psi_{2r} = \psi_{3r}$). This F-test can be computed using the `sirt::mcmc_WaldTest()` function. Seven out of ten raters showed significant differences in item-specific rater severities while for no rater the F-test of the equality of rater variances was significant.

[2] Although much more iterations than in the estimation of the GMFRM were chosen, computation time did not substantially increase because the MCMC algorithm in **immer** is implemented in R using the **Rcpp** package for some parts of the computation.

6 Discussion

In the past sections, we gave an overview of opportunities in psychometric modeling in the field of rater studies and we provided some insight into popular estimation methods. We have introduced models ranging from G-theory, Rasch-MFRM to more recent developments such as hierarchical modeling approaches (GMFRM or HRM). Several basic considerations of assumptions, expectations and properties of the models which are all associated with model choice have been elaborated in Section 4. To take dependencies into account, either between persons and items or persons and raters, the HRM (in the first case) or the GMFRM (for both cases) might be considered. As stated, a drawback might be that the person ability has to be interpreted as conditioned to the modeled dependence. Sometimes it might be more appropriate to treat those dependencies as nuisance factors, in particular, when using the sum scores and the equal weighting of items and raters is favored. In this case, the Rasch-MFRM or G-theory models might be appropriate choices.

To gain an impression which psychometric models are applied in the field of language testing, we conducted a rough literature study. For this study we have used two journals "Language Testing" and "Language Assessment Quarterly". All contributions available online between 2007 and 2017, which have dealt with rater studies, were taken into account. The applied methods were classified into three groups, the Rasch-MFRM, G-theory and a remaining third category "other". The latter category includes both qualitative and quantitative analysis like descriptive statistics, as well as more complex models, such as structural equation models, generalized linear models, etc. It appeared that the Rasch-MFRM is currently the favored model for rater studies within these two selected journals. Over the last 10 years, the Rasch-MFRM has gained popularity. Between 2007 and 2017 the Rasch-MFRM was used in 51.5%, the G-theory in 19.1%, and the "other" methods in 29.4% of the cases. Although this study is not representative for applied methods within the field of language testing, it becomes apparent that there is a considerable preference for the Rasch-MFRM. Similarly, McNamara and Knoch (2012) reviewed the usage of IRT model in the field of language testing between 1984 and 2002 and found that the Rasch model was dominantly used. The authors concluded that development in psychometric methods creates many opportunities, but is also related to challenges of its application by language testers because the interpretation of more complex models is involved.

We think that in the near future more advanced methodological developments will be applied because of a wider availability of software and an increasing familiarity of researchers with recent software. We hope that this paper as well as the fast growing community of package development in R will contribute to reduce the still existing gap between the methodological developments on the one hand and the variety of methods in the field of language testing on the other hand.

Some aspects have yet not been addressed in this paper, but may be also influencing factors for model selection in the broadest sense. First of all, the choice of a rating design

should be emphasized. When choosing a rating design, there are a few possibilities which reach from complete designs in which every rater judges every item to more sophisticated incomplete designs which usually requires a kind of linking. Complete rating designs are often not applicable for economic reasons that is why incomplete designs are chosen. Several types of incomplete rating designs are possible; one of the more established ones might be the practice of using common ratings. Here, a representative selection of persons is rated by all raters, while the remaining ones are rated by one or more, but not by all raters. Another possibility of rating designs which might reduce the venture of choosing not representative common persons are overlap (or incomplete) designs (e.g. DeCarlo, 2010). Different types of overlap designs are feasible. All of them have in common that decisions are required, which persons are allocated to rater, how many persons are rated per rater, and how raters are linked among each other. These considerations or decisions can themselves lead to additional effects (Casabianca & Wolfe, 2017).

Another yet not mentioned aspect of rater models is the development of automated scoring systems, especially in the area of large-scale assessments. From an economical point of view, this approach might be more efficient than human ratings although the validity of automated scoring approaches can be questioned. It should be noted that the discussion about choosing appropriate rater models for human raters is also important for calibrating automated scoring systems because the calibration relies on human ratings (Wind, Wolfe, Engelhard, Foltz, & Rosenstein, 2018).

Acknowledgment

We would like thank Thomas Eckes for stimulating us to contribute to this special issue, and in particular for his consideration and very helpful feedback on previous versions of the manuscript.

References

Aitkin, M. (2016). Expectation maximization algorithm and extensions. In W. van der Linden (Ed.), *Handbook of item response theory. Volume Two: Statistical Tools* (pp. 217–236). Boca Raton: CRC Press.

Andersen, E. B. (1980). *Discrete statistical models with social science applications*. Amsterdam: North-Holland Publishing Co.

Andrich, D. (1978). A rating formulation for ordered response categories. *Psychometrika*, *43*(4), 561–573.

Arminger, G., & Schoenberg, R. J. (1989). Pseudo maximum likelihood estimation and a test for misspecification in mean and covariance structure models. *Psychometrika*, *54*(3), 409–425.

Bates, D., Mächler, M., Bolker, B., & Walker, S. (2015). Fitting linear mixed-effects models using lme4. *Journal of Statistical Software*, *67*(1), 1–48.

Bechger, T. M., Maris, G., & Hsiao, Y. P. (2010). Detecting halo effects in performance-based examinations. *Applied Psychological Measurement*, *34*(8), 607–619.

Berk, R., Brown, L., Buja, A., George, E., Pitkin, E., Zhang, K., & Zhao, L. (2014). Misspecified mean function regression: making good use of regression models that are wrong. *Sociological Methods & Research*, *43*(3), 422–451.

Bertoli-Barsotti, L., Lando, T., & Punzo, A. (2014). Estimating a Rasch model via fuzzy empirical probability functions. In D. Vicari, A. Okada, G. Ragozini, & C. Weihs (Eds.), *Analysis and Modeling of Complex Data in Behavioral and Social Sciences* (pp. 29–36). Cham, Switzerland: Springer.

Bock, R. D., Brennan, R. L., & Muraki, E. (2002). The information in multiple ratings. *Applied Psychological Measurement*, *26*(4), 364–375.

Bollen, K. A. (1989). *Structural equations with latent variables*. New York: Wiley.

Bond, T. G., & Fox, C. M. (2001). *Applying the Rasch model: Fundamental measurement in the human sciences*. Mahwah: Lawrence Erlbaum.

Brennan, R. L. (2001a). *Generalizability theory*. New York: Spinger.

Brennan, R. L. (2001b). Some problems, pitfalls, and paradoxes in educational measurement. *Educational Measurement: Issues and Practice*, *20*(4), 6–18.

Brennan, R. L. (2011). Generalizability theory and classical test theory. *Applied Measurement in Education*, *24*(1), 1–21.

Casabianca, J. M., & Wolfe, E. W. (2017). The impact of design decisions on measurement accuracy demonstrated using the hierarchical rater model. *Psychological Test and Assessment Modeling*, *59*(4), 471–492.

De Boeck, P. (2008). Random item IRT models. *Psychometrika*, *73*(4), 533–559.

DeCarlo, L. T. (2005). A model of rater behavior in essay grading based on signal detection theory. *Journal of Educational Measurement*, *42*(1), 53–76.

DeCarlo, L. T. (2010). *Studies of a latent class signal detection model for constructed response scoring II: Incomplete and hierarchical designs* (ETS Research Report RR-10-08). Princeton: ETS.

DeCarlo, L. T., Kim, Y., & Johnson, M. S. (2011). A hierarchical rater model for constructed responses, with a signal detection rater model. *Journal of Educational Measurement*, *48*(3), 333–356.

Eckes, T. (2005). Examining rater effects in TestDaF writing and speaking performance assessments: A many-facet Rasch analysis. *Language Assessment Quarterly*, *2*(3), 197–221.

Eckes, T. (2015). *Introduction to many-facet Rasch measurement: Analyzing and evaluating rater-mediated assessments (2nd ed.)*. Frankfurt am Main: Peter Lang.

Eckes, T. (2017). Guest Editorial Rater effects: Advances in item response modeling of human ratings–Part I. *Psychological Test and Assessment Modeling*, *59*(4), 443–452.

Embretson, S. E. (1999). Generating items during testing: Psychometric issues and models. *Psychometrika*, *64*(4), 407–433.

Engelhard, G. (1992). The measurement of writing ability with a many-faceted Rasch model. *Applied Measurement in Education*, *5*(3), 171–191.

Engelhard, G. (1994). Examining rater errors in the assessment of written composition with a many-faceted Rasch model. *Journal of Educational Measurement*, *31*(2), 93–112.

Fischer, G. H. (1973). The linear logistic test model as an instrument in educational research. *Acta Psychologica*, *37*(6), 359–374.

Fischer, G. H. (1995). Derivations of the Rasch model. In G. H. Fischer & I. W. Molenaar (Eds.), *Rasch models. foundations, recent developments, and applications* (pp. 15–31). New York: Springer.

Forero, C. G., & Maydeu-Olivares, A. (2009). Estimation of IRT graded response models: Limited versus full information methods. *Psychological Methods*, *14*(3), 275–299.

Formann, A. K. (1985). Constrained latent class models: Theory and applications. *British Journal of Mathematical and Statistical Psychology*, *38*(1), 87–111.

Garner, M., & Engelhard, G. (2009). Using paired comparison matrices to estimate parameters of the partial credit Rasch measurement model for rater-mediated assessments. *Journal of Applied Measurement*, *10*(1), 30–41.

Hahn, J., & Newey, W. (2004). Jackknife and analytical bias reduction for nonlinear panel models. *Econometrica*, *72*(4), 1295–1319.

Holland, P. W. (1990). On the sampling theory foundations of item response theory models. *Psychometrika*, *55*(4), 577–601.

Ip, E. H. (2010). Empirically indistinguishable multidimensional IRT and locally dependent unidimensional item response models. *British Journal of Mathematical and Statistical Psychology*, *63*(2), 395–416.

Katsikatsou, M., Moustaki, I., Yang-Wallentin, F., & Jöreskog, K. G. (2012). Pairwise likelihood estimation for factor analysis models with ordinal data. *Computational Statistics & Data Analysis*, *56*(12), 4243–4258.

Leckie, G., & Baird, J.-A. (2011). Rater effects on essay scoring: A multilevel analysis of severity drift, central tendency, and rater experience. *Journal of Educational Measurement*, *48*(4), 399–418.

Linacre, J. (1989). *Many-facet Rasch measurement.* Chicago, IL: MESA Press.

Linacre, J. (2017). *A user's guide to FACETS: Rasch-model computer programs.* Chicago: Winsteps.com.

Lord, F. M. (1980). *Applications of item response theory to practical testing problems*. Mahwah, New Jersey: Erlbaum.

Luecht, R., & Ackerman, T. A. (2018). A technical note on IRT simulation studies: Dealing with truth, estimates, observed data, and residuals. *Educational Measurement: Issues and*

Practice, Advance online publication. doi: 10.1111/emip.12185

Lunz, M. E., Wright, B. D., & Linacre, J. M. (1990). Measuring the impact of judge severity on examination scores. *Applied Measurement in Education*, *3*(4), 331–345.

Mair, P., & Hatzinger, R. (2007). Extended Rasch modeling: The eRm package for the application of IRT models in R. *Journal of Statistical Software*, *20*(9), 1–20.

Masters, G. N. (1982). A Rasch model for partial credit scoring. *Psychometrika*, *47*(2), 149–174.

McLaughlin, J. E., Singer, D., & Cox, W. C. (2017). Candidate evaluation using targeted construct assessment in the multiple mini-interview: A multifaceted Rasch model analysis. *Teaching and Learning in Medicine*, *29*(1), 68–74.

McNamara, T., & Knoch, U. (2012). The Rasch wars: The emergence of Rasch measurement in language testing. *Language Testing*, *29*(4), 555–576.

Muraki, E. (1992). A generalized partial credit model: Application of an EM algorithm. *Applied Psychological Measurement*, *16*(1), 159–176.

Muthén, B. (1984). A general structural equation model with dichotomous, ordered categorical, and continuous latent variable indicators. *Psychometrika*, *49*(1), 115–132.

Muthén, B., & Asparouhov, T. (2012). Bayesian structural equation modeling: A more flexible representation of substantive theory. *Psychological Methods*, *17*(3), 313–335.

Myford, C. M., & Wolfe, E. W. (2003). Detecting and measuring rater effects using many-facet Rasch measurement: Part I. *Journal of Applied Measurement*, *4*(4), 386–422.

Patz, R. J., & Junker, B. W. (1999). Applications and extensions of MCMC in IRT: Multiple item types, missing data, and rated responses. *Journal of Educational and Behavioral Statistics*, *24*(4), 342–366.

Patz, R. J., Junker, B. W., Johnson, M. S., & Mariano, L. T. (2002). The hierarchical rater model for rated test items and its application to large-scale educational assessment data. *Journal of Educational and Behavioral Statistics*, *27*(4), 341–384.

Plummer, M. (2003). *JAGS: A program for analysis of Bayesian graphical models using Gibbs sampling*. Vienna,Austria: Proceedings of the 3rd International Workshop on Distributed Statistical Computing (DSC 2003), March 20–22.

R Core Team. (2018). *R: A language and environment for statistical computing.* Austria: R foundation for statistical computing Vienna.

Reckase, M. D. (2017). *A tale of two models: Sources of confusion in achievement testing*. (Research Report No. RR-17-44). Princeton, NJ: Educational Testing Service.

Robitzsch, A. (2018a). *LAM: Some latent variable models*. R package version 0.2. https://CRAN.R-project.org/package=LAM.

Robitzsch, A. (2018b). *sirt: Supplementary item response theory models*. R package version 2.5. https://CRAN.R-project.org/package=sirt.

Robitzsch, A., Kiefer, T., & Wu, M. (2018). *TAM: Test analysis modules*. R package version 2.9.

https://CRAN.R-project.org/package=TAM.

Robitzsch, A., & Steinfeld, J. (2018). *immer: Item response models for multiple ratings*. R package version 1.0. https://CRAN.R-project.org/package=immer.

Rosseel, Y. (2012). lavaan: An R Package for Structural Equation Modeling. *Journal of Statistical Software*, *48*(2), 1–36.

Rost, J., & Langeheine, R. (1997). A guide through latent structure models for categorical data. In J. Rost & R. Langeheine (Eds.), *Applications of latent trait and latent class models in the social sciences* (pp. 13–37). New York: Waxmann.

Rusch, T., Mair, P., & Hatzinger, R. (2013). *Psychometrics with R: A review of CRAN packages for item response theory*. WU Vienna University of Economics and Business.

Saal, F. E., Downey, R. G., & Lahey, M. A. (1980). Rating the ratings: Assessing the psychometric quality of rating data. *Psychological Bulletin*, *88*(2), 413–428.

Samejima, F. (1969). *Estimation of ability using a response pattern of graded scores*. Psychometrika Monograph No. 17. Richmond, VA: Psychometric Society.

Tor, E., & Steketee, C. (2011). Rasch analysis on OSCE Data: An illustrative example. *The Australasian Medical Journal*, *4*(6), 339.

Tuerlinckx, F., & De Boeck, P. (2004). Models for residual dependencies. In P. De Boeck & M. Wilson (Eds.), *Explanatory item response models: A generalized linear and nonlinear approach* (pp. 289–316). New York: Springer.

Tutz, G., & Schauberger, G. (2015). A penalty approach to differential item functioning in Rasch models. *Psychometrika*, *80*(1), 21–43.

van der Linden, W. J. (1994). Fundamental measurement and the fundamentals of Rasch measurement. In M. Wilson (Ed.), *Objective measurement: Theory into practice Vol. 2* (pp. 3–24). Norwood, NJ: Ablex Publishing Cooperation.

Varin, C., Reid, N., & Firth, D. (2011). An overview of composite likelihood methods. *Statistica Sinica*, *21*, 5–42.

Verhelst, N. D., & Verstralen, H. H. (2001). An IRT model for multiple raters. In A. Boomsma, M. A. J. van Duijn, & T. A. B. Snijders (Eds.), *Essays on item response theory* (pp. 89–108). New York: Springer.

Wang, W.-C., Su, C.-M., & Qiu, X.-L. (2014). Item response models for local dependence among multiple ratings. *Journal of Educational Measurement*, *51*(3), 260–280.

Warm, T. A. (1989). Weighted likelihood estimation of ability in item response theory. *Psychometrika*, *54*(3), 427–450.

White, H. (1982). Maximum likelihood estimation of misspecified models. *Econometrica: Journal of the Econometric Society*, *50*, 1–25.

Wind, S. A., & Peterson, M. E. (2018). A systematic review of methods for evaluating rating quality in language assessment. *Language Testing*, *35*(2), 161–192.

Wind, S. A., Wolfe, E. W., Engelhard, G. J., Foltz, P., & Rosenstein, M. (2018). The influence of rater effects in training sets on the psychometric quality of automated scoring for writing assessments. *International Journal of Testing*, *18*(1), 27–49.

Wolfe, E. W. (2014). *Methods for monitoring rating quality: Current practices and suggested changes*. White Paper. Pearson Research Reports, Pearson.

Wu, M. (2017). Some IRT-based analyses for interpreting rater effects. *Psychological Test and Assessment Modeling*, *79*(4), 453–470.

Wu, S. M., & Tan, S. (2016). Managing rater effects through the use of FACETS analysis: the case of a university placement test. *Higher Education Research & Development*, *35*(2), 380–394.

Yuan, K.-H., & Schuster, C. (2013). Overview of statistical estimation methods. In T. D. Little (Ed.), *The Oxford Handbook of Quantitative Methods* (Vol. 1, pp. 361–387). Oxford: Oxford University Press.

Zwinderman, A. H. (1995). Pairwise parameter estimation in Rasch models. *Applied Psychological Measurement*, *19*(4), 369–375.